The Role of Financial Institutions in An Economic Growth Model with Renewable Natural Resource and Endogenous Technology

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ABSTRACT

This thesis presents a three-sector finance-extended endogenous growth model with constant returns to scale in renewable natural resource production in combination with physical and technological capital. The purpose of this thesis is to provide a theoretical framework that investigates whether and how financial institutions impact capital accumulation, output productivity and economic growth through the channel of renewable natural capital. Sound financial institutions improve savings and investments and also effectively allocate resources in capital producing ventures that in return enhance output productivity and stimulate economic growth. In this model, renewable natural capital will be used in the production of the final consumption good and the technological capital. I will solve the model along balanced growth path (BGP) and further discuss stability analysis and transitional dynamics of my model. I have found that renewable natural capital and technological capital accumulation positively depend on financial development therefore developing economies with a well developed financial sector display higher output growth and reach to the global frontier at a faster rate.

DECLARATION

I hereby declare that this thesis is my own work. It is being submitted to Lahore School of Economics for the completion of the Degree of M.Phil Economics. It has not been submitted before for any degree or examination at any other institute.

Nayab Kanwal

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INTRODUCTION

In economic growth literature, factor that result in the differences in countries' growth rates has always been a core question. Technology is the mainspring of long run economic growth and therefore it is a fundamental factor for countries' growth differences [Aghion and Howitt (2005)]. Both Klenow and Rodriguez-Clare (1997) and Easterly and Levine (2001) empirically estimated growth rates differences and concluded that most of the variation in countries' growth rates is attributable to differences in total factor productivity rather than factor accumulation. Easterly and Levine (2001) further concluded that countries tend to have stable factor accumulation but still vary in long run growth because of technology that cause differences in productivity growth. However according to Aghion et al. (2005), the differences in productivity growth are not only caused by technology but also because of other factors that include: institutions and geographical location. Empirically, it can be concluded that differences in countries' growth rates are due to differences in productivity growth caused by three main factors: technology, institutions and geographical location. The purpose of this thesis is therefore to build an endogenous growth model that incorporates these factors. Financial institutions are the most important channel through which resources are directed towards technological intensive production that stimulates productivity growth [Aghion et al. (2005); Aghion and Howitt (2008); Agn (2011); Ilyina and Samaniego (2011)].

In empirical literature, financial institutions and economic growth have a puzzling relationship. For the purpose of examining the impact of finance on economic growth, financial institutions are often characterized in two broad categories. Firstly, financial institutions as the *facilitator* which performs intermediation activities and secondly financial institutions as the *financial center* which performs both intermediation and non-intermediation activities [Beck et al. (2014)]. According to pioneer work by Pagano (1993), financial intermediaries contribute towards more effective distribution of resources and hence promote economic growth. In this regard, financial intermediaries increase the economy's saving rate as a whole and further channel those saving towards more effective ventures. All these activities contribute towards economic growth through two broad channels; capital accumulation and technological advancements [Levine (1997)]. Firstly, the financial intermediaries increase the rate of capital accumulation either by increasing economy's whole saving rates or by channeling those savings among productive capital producing ventures [Trew (2014)] and this channel is linked with the efficiency of investment [Nili and Rastad (2007)].

Secondly financial intermediaries mobilize saving effectively and simultaneously increase investments in the economy that in result stimulates economic growth by improving the rate of technological advancements [Romer (1990); Aghion and Howitt (1992)]. Likewise, in the empirical literature many papers identified the positive relationship between financial development and physical capital accumulation through the channel of effective allocation of resources and were therefore able to predict subsequent economic growth [King and Levine (1993b); Levine (2005); Yuxiang and Chen (2011); Beck (2012)].

A large body of literature has now raised concerned towards the benefits of finance that weather there is a limit to the financial development. According to recent empirical literature, a non-linear relationship may exist between finance and economic growth indicators. Masten et al. (2008) empirically investigate the relationship of financial development and economic growth in Europe by modeling threshold effect of financial development. They found that the effect of finance approaches zero after a threshold level and below that threshold level the effect of finance on economic growth is the largest. Similarly, Rousseau and Wachtel (2011) empirically tested finance-growth relationship for 84 countries where they found that financial development has a vanishing effect on countries economic growth rates. Furthermore, many empirical papers suggested an inverse U-shaped relationship between finance and economic growth, as they estimated that financial development is negatively correlated with economic growth indicators after reaching a threshold level of financial deepening [Arcand et al. (2015); Aizenman et al. (2015); Samargandi (2015); Soedarmono et al. (2017); Bucci and Marsiglio (2018)]. Recently Bucci and Marsiglio (2018) has also suggested some precise functional forms of this non-monotonic finance-growth relationship.

From above discussion it is clear that the effect of financial development on economic growth is still ambiguous. However many papers have tried to find an appropriate reasoning for this relationship. Aghion et al. (2005) argued that the effect of financial development diminishes as economy moves towards the global frontier, as the main function of financial intermediaries is to enhance productivity growth, therefore the effect of financial development is limited for economies closer to the global frontier. And they referred it as a main factor of the non-linear relationship of the finance-growth. However the methods used for measuring financial development also have a critical role in explaining the finance-growth relationship. King and Levine (1993a) formulated different measures of financial development and later Levine (2005) discussed those measures in detail. The most important measures of financial development are; *depth*, *bank*, *privy* and *turnover ratio*[1] [King and Levine (1993a); Levine and Zervos (1998a)]. In empirical literature, the monotonic

[1] *depth* = Total liquid liabilities/GDP, *bank* = Bank credit/(Bank credit + Central bank domestic assets), *privy* = Credit to private enterprizes/GDP, *turnover ratio* = Total value of shares traded/Stock market capitalization.

finance-growth relationship is estimated by using *depth* and *privy* [King and Levine (1993a); Beck et al. (2000); Rioja and Valev (2004)]. However, Arcand et al. (2015) found that the non-monotonic relationship between financial development (using *depth* and *privy*) and economic growth exists after controlling for *turnover ratio*.

In theoretical literature, Bucci and Marsiglio (2018) found non-monotonic relationship between financial development and economic growth by using physical capital and human capital. According to their finding, finance tends to have an adverse effect on economic growth after a threshold level where the productivity effect of human capital is less than the depreciation effect of human capital. Finance is beneficial as far as productivity effect dominates the depreciation effect. However it is critical to note that the non-monotonic relationship exists only under specific functional forms of productivity and depreciation of human capital. As the monotonic relationship exists under linear functional forms of productivity and depreciation of human capital and non-monotonic under exponential and quadratic forms of productivity and depreciation of human capital.

The finance-growth relationship is quite complex in nature both empirically and theoretically, while it is scarcely found in theoretical literature. Therefore the aim of this thesis is to theoretically investigate the role of financial institutions in capital accumulation, output productivity and economic growth. In this regard, I will build a three-sector endogenous growth model with physical capital, renewable natural capital and technology where financial institutions affect all three sectors.

The United Nations defined natural resources as "natural assets occurring in nature that can be used for economic production or consumption" [United Nations (1997)]. They are further divided into two broad categories: non-renewable and renewable natural resources. Non-renewable are exhaustible natural resources such as fossil fuels that cannot be regenerated after exploitation. On other hand, renewable are inexhaustible natural resources that are naturally replenishing because of natural recurring process [United Nations (1997)]. Solar, wind, biomass, water (hydropower) and geothermal are the five major types of renewable natural resources.

In natural resource economics, there has been a debate over resource endowments and economic growth results. The most cited paper is by Sachs and Warner (1995) where they documented an inverse relationship between natural resource endowment and economic growth in 97 countries. Likewise many other studies concluded the same results [Auty (2001); Gylfason (2001); Robinson et al. (2006)]. On other hand, natural resource abundance deters human and social capital growth through indirect crowding-out effect. Resource-rich economies have weak political and financial institutions due to rent seeking and neglected education, for that reason these economies grow slower than resource-poor economies [Auty (2001); Torvik (2002); Wadho (2014)]. Economists therefore argued institutions are one of the most important determinant of economic growth, as countries with strong institutions benefit from resource booms, such as USA, Norway and Botswana [Gylfason (2001); Stijns (2006); Auty (2007); Zubikova (2018)]. Strong financial institutions are the backbone of these success stories, by its function of allocating savings and investments productively [Gylfason and Zoega (2006); Nili and Rastad (2007)]. Moreover, developed financial institutions lower resource rents through better monitoring mechanism, hence improving financial institutions is the baseline for stimulating economic growth [Shahbaz et al. (2018)].

In terms of renewable natural resources financial institutions play the most important role. As, the main obstacle in the deployment of renewable natural resources is the financing due to its high upfront capital cost in combination with high information cost [Brunnschweiler (2010); Kim and Park (2016)]. Financial development positively contribute to the accumulation of renewable natural capital through easing the financing constraints [Brunnschweiler

(2010); Kim and Park (2016)]. The effect of financial development becomes stronger in sectors that heavily rely on external financing as suggested by Rajan and Zingales (1998). However in this regard, Kim and Park (2016) found that from the external finance-dependent sectors financial development effect may be more prevalent in renewable sectors than industrial sectors.

Most of the empirical work has investigated the relationship between natural resources and economic growth by assuming non-renewable natural resources. Few recent studies have tested the relationship of renewable energy consumption with economic growth and found a significant positive relationship [Bhattacharya et al. (2016); Armeanu et al. (2017); Rafindadi and Ozturk (2017)]. Similarly, models of economic growth and renewable natural resources are also found scarcely in theoretical literature, as the main focus was on non-renewable natural resources. Some articles have been filling the gaps by considering continuous time models and these growth models are developed in general equilibrium context [Li and Lofgren (2000); Ayong Le Kama (2001); Eliasson and Turnovsky (2004); Wirl (2004)]. Aznar-Marquez and Ruiz-Tamarit (2005) considered an endogenous growth model, similar to the one formulated by Lucas (1988), with constant returns to scale in renewable resource production in combination with physical capital. Russu (2012) extended their model by adding environmental externalities.

Besides financial institutions and renewable natural capital, technology is also an important feature of this thesis. In the theoretical economics literature, endogenous growth models were based on technological advancements and therefore in those models technology is the mainspring of long run economic growth. Such as Romer (1990), Aghion and Howitt (1992), Barbier (1999), Scholz and Ziemes (1999), Jones (2005) and Bretschger and Smulders (2012) developed endogenous growth models where technology is endogenously determined by innovation and long-run equilibria are achieved by accumulating more technological capital. Valente (2010) referred technology accumulation as the engine of growth as it helps economies to build backstop technology in the form of renewable natural resources. Therefore, technological advancement plays a vital role in the accumulation of renewable natural resource capital.

In regards to endogenous technology, Aghion and Howitt (1998) developed an endogenous technology model by incorporating a non-renewable resource to the AK-model and to the Schumpeterian approach. By using their approach, Grimaud and Roug (2003) and Groth (2005) included a non-renewable natural resource in the production of final good as well as an input to the innovation sector. Later, Bretschger and Smulders (2012) formulated an endogenous multi-sector growth model with non-renewable natural resources and concluded that sustainable long run growth rely on development of innovations and the profitability of R&D investments. None of the economic growth models have incorporated renewable natural resource in endogenous technology model.

In my model, the role of financial institutions will be studied where I have considered financial institutions as the *facilitator* that performs intermediation activities. Financial intermediaries affect all three sectors; renewable natural resource, physical capital and endogenous technology. In the theoretical literature, Aghion et al. (2005) developed discrete time framework to investigate the relationship between financial development and economic growth by taking endogenous technology. Whereas in economic growth literature, no one has considered financial development and endogenous technology in continuous time framework. The important contribution of my thesis is therefore that I have modeled a three-sector endogenous growth model that investigates the role of financial institutions and endogenous technology in continuous time framework.

This thesis takes Aznar-Marquez and Ruiz-Tamarit (2005) two sector economic growth model, with physical capital and renewable natural resources, as a baseline model by incorporating the financial institutions and endogenous technology with no externality case. Bucci and Marsiglio (2018) incorporated financial institutions by formulating Uzawa (1965)-Lucas (1988) type growth model in combination with human and physical capital. In Bucci and Marsiglio (2018) model, financial development affect steady state growth by altering human and physical capital accumulation in continuous time framework and such work has not been previously done in literature. The financial development is incorporated into my model by following Bucci and Marsiglio (2018) idea. Whereas, the endogenous technology is incorporated by following La Torre and Marsiglio (2010) idea, where they developed three-sector economic growth model with physical capital, human capital and endogenous technology.

The model will be solved in standard mathematical procedures for BGP equilibrium and steady state values, as is done in the literature as well. This thesis is divided into four chapters. Chapter 1 will comprise the model formulation and the necessary and sufficient conditions of optimal control. Chapter 2 will comprise determination of BGP and I will discuss the role of financial development on economic growth and BGP equilibrium by considering different functional forms. Later, the BGP equilibrium will be analyzed at benchmark values. I will perform a local stability analysis in chapter 3. In chapter 4, the effect of financial development on BGP equilibrium will be studied and further numerical simulations by analyzing the effect of change of different parameters on BGP equilibrium. In the end I will summarize with concluding remarks.

Chapter 1

Financial development, natural resources, endogenous technology and economic growth

In this chapter I will present a three-sector finance extended endogenous growth model with renewable natural capital and physical capital. I will check sufficiency conditions through Mangasarian and Arrow's sufficiency theorem as existence of BGP is guaranteed provided sufficient conditions hold.

1.1 Framework of the model

This thesis develops a Lucas (1988) type three-sector finance-extended endogenous growth model with constant returns to scale in natural resource production in combination with physical capital and endogenous technology. Aznar-Marquez and Ruiz-Tamarit (2005), Bucci and Marsiglio (2018) and La Torre and Marsiglio (2010) are the baseline models of my framework.

In this model, renewable natural capital will be used in the production of the final consumption good and the technological capital. In order to maximize the utility, the model uses a current value Hamiltonian which is maximized subject to three constraints; physical capital, natural resource capital and technological capital constraints.

1.1.1 Production

The Cobb-Douglas production function is

$$Y(t) = (A(t)L(t))^{1-\alpha-\beta}K(t)^{\alpha}(z(t)Q(t))^{\beta} \tag{1.1}$$

where $Y(t)$ is the output, the variable $K(t)$ is the level of physical capital, $Q(t)$ is the renewable natural capital and $z(t)$ is the aggregate extraction rate of renewable natural resource used in the production of final good with $0 < z(t) < 1$. Here, α is the share of physical capital, and the share of renewable natural capital is denoted by β, with the condition that $\alpha + \beta < 1$. Production, $Y(t)$, depends positively on stock of physical capital $K(t)$ and stock of extracted renewable natural capital $z(t)Q(t)$. Lastly, $A(t)$ is existing stock of technology in the form of new ideas. The model uses Aznar-Marquez and Ruiz-Tamarit (2005) methodology to incorporate renewable natural capital in production function.[1] Similarly the model incorporates technological capital in production function by using La Torre and Marsiglio (2010) approach.

In this model, the population is constant therefore the population growth rate is normalized to one. Hence the production function in per capita terms is as follows [La Torre and Marsiglio (2010)]:

$$\frac{Y(t)}{L(t)} = \frac{(A(t)L(t))^{1-\alpha-\beta}K(t)^{\alpha}(z(t)Q(t))^{\beta}}{L(t)} \tag{1.2}$$

$$y(t) = A(t)^{1-\alpha-\beta}L(t)^{-\alpha-\beta}K(t)^{\alpha}(z(t)Q(t))^{\beta} \tag{1.3}$$

$$y(t) = A(t)^{1-\alpha-\beta}k(t)^{\alpha}(z(t)q(t))^{\beta} \tag{1.4}$$

[1] Countries with low level of renewable natural capital have other factors of production to compensate, as they are either more labor-intensive or capital-intensive.

1.1.2 Utility function

The representative agent's utility maximization problem can be rewritten in per capita as follows:

$$\text{Max}_{c,z,x} \quad \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}, \ \sigma \neq 1 \tag{1.5}$$

where $c(t)$ is consumption per capita, σ is the inverse of constant intertemporal elasticity of substitution and ρ is the discount factor, with $\rho > 0$.

1.1.3 Physical capital

The standard law of motion for the stock of physical capital is

$$\dot{K}(t) = I(t) - \delta K(t) \tag{1.6}$$

where $I(t)$ is investment and $0 < \delta < 1$ is the depreciation rate of physical capital. I can write equation (1.6) in capita terms as follows:

$$\frac{\dot{K}(t)}{L(t)} = \frac{I(t) - \delta K(t)}{L(t)} \tag{1.7}$$

as we know that

$$k(t) = \frac{K(t)}{L(t)} \tag{1.8}$$

$$\dot{k}(t) = \frac{\dot{K}(t)}{L(t)} - \frac{K(t)}{L(t)^2}\dot{L}(t) \tag{1.9}$$

$$\dot{k}(t) = \frac{\dot{K}(t)}{L(t)} - k(t)\frac{\dot{L}(t)}{L(t)} \tag{1.10}$$

In this model, the population is constant therefore the population growth rate is normalized to one ($L(t) = 1$, $\frac{\dot{L}(t)}{L(t)} = 0$), so

$$\dot{k}(t) = \frac{\dot{K}(t)}{L(t)} \tag{1.11}$$

hence equation (1.7) can be written as

$$\frac{\dot{K}(t)}{L(t)} = \dot{k}(t) = i(t) - \delta k(t) \tag{1.12}$$

The resource constraint is:

$$Y(t) = C(t) + I(t) \tag{1.13}$$

By using resource constraint, economy's investment function can be written as:

$$I(t) = Y(t) - C(t) \tag{1.14}$$

Equation (1.14) can also be written in per capita terms as follows:

$$\frac{I(t)}{L(t)} = \frac{Y(t)}{L(t)} - \frac{C(t)}{L(t)} \tag{1.15}$$

$$i(t) = y(t) - c(t) \tag{1.16}$$

Financial institutions have been incorporated in the model in the form of financial intermediaries by following Bucci and Marsiglio (2018). Financial intermediaries charge fee for the supply of financial service, hence they absorb a share of resources from physical capital accumulation [Bencivenga and Smith (1991); Pagano (1993); Bucci and Marsiglio (2018)]. The share of resources that will be subtracted from physical capital accumulation is $\xi(\phi)$ where (ϕ) denotes the degree of financial development. $\xi(\phi)$ lies between zero and one depending on the degree of financial development ϕ, higher the value of ϕ means higher the financial development. Therefore less resources will be wasted in the process of financial intermediation when the value of $\xi(\phi)$ is close to zero [Bucci and Marsiglio (2018)].

After considering financial institutions in resource constraint, economy's investment function can be written as:

$$i(t) = [1 - \xi(\phi)]y(t) - c(t) \tag{1.17}$$

By plugging value of $y(t)$ from equation (1.4), equation (1.17) can be rewritten as:

$$i(t) = [1 - \xi(\phi)]A(t)^{1-\alpha-\beta}k(t)^{\alpha}(z(t)q(t))^{\beta} - c(t) \tag{1.18}$$

By combining equations (1.12) and (1.18), the final equation of physical capital accumulation is given by:

$$\dot{k}(t) = [1 - \xi(\phi)]A(t)^{1-\alpha-\beta}k(t)^{\alpha}(z(t)q(t))^{\beta} - c(t) - \delta k(t) \tag{1.19}$$

The final equation of motion for physical capital is the difference of net of per capita output, obtained after deducting depreciation cost and fee of financial services, and per capita consumption [Bucci and Marsiglio (2018)].

1.1.4 Renewable Natural capital

In this model renewable natural resource is considered as capital, that is inexhaustible naturally replenishing natural resources because of natural recurring process. Further, there are some assumptions on renewable natural resources (similar to Aznar-Marquez and Ruiz-Tamarit (2005); firstly renewable natural capital satisfies the properties of rivalry and excludability. Secondly, property rights of renewable natural resources are equally and uniformly distributed. Lastly, the model considers private property rights system in order to avoid tragedy of common property or open access regime. As economy collapses in the presence of open access regime, due to inefficient over-exploitation of natural resources [Brander and Taylor (1998)].

Stock of renewable natural capital, $Q(t)$, is taken as an input in the production of the final consumption good as well as in the production of technological capital. $x(t)$ is the aggregate extraction rate of natural resources in production of technological capital, with $0 < x(t) < 1$. Therefore, the natural resource extraction for the production of final good and production of technological capital is

$$\textit{Total extraction} = z(t)Q(t) + x(t)Q(t) \tag{1.20}$$

$z(t) + x(t)$ is the harvested part on renewable natural capital with $z(t) + x(t) < 1$. And $1 - z(t) - x(t)$ is the share of remaining renewable natural capital, therefore $(1 - z(t) - x(t))Q(t)$ is the remaining stock of renewable natural capital. In renewable natural resource sector, the main obstacle in the extraction of renewable natural resources is the financing due to its high upfront capital cost in combination with high information cost [Brunnschweiler (2010); Kim and Park (2016)]. Financial development positively contribute to the accumulation of renewable natural capital through easing the financing constraints [Brunnschweiler (2010); Kim and Park (2016)]. Therefore, sound financial systems ease the process of development of renewable energy sources, which increase the natural capital stock in the economy [Ba et al. (2010); Calitz and Fourie (2010); Mathews et al. (2010); Scholtens and Veldhuis (2015); Mazzucato and Semieniuk (2018)]. Thus renewable natural capital growth rate $B(\phi)$, is a positive function of ϕ , that determines the degree of the development of financial sector $\phi > 0$. Therefore the current stock of renewable natural capital is given by:

$$\textit{Current stock of renewable natural capital} = B(\phi)(1-z(t)-x(t))Q(t) \quad (1.21)$$

Finally the equation of motion for natural capital is the difference between current stock of renewable natural capital and total extraction (by taking values from equations (1.20) and (1.21) [Aznar-Marquez and Ruiz-Tamarit (2005)]:

$$\dot{Q}(t) = B(\phi)(1 - z(t) - x(t))Q(t) - z(t)Q(t) - x(t)Q(t) \quad (1.22)$$

This can be rewritten in per capita terms as follows:

$$\frac{\dot{Q}(t)}{L(t)} = \frac{B(\phi)(1 - z(t) - x(t))Q(t)}{L(t)} - \frac{z(t)Q(t)}{L(t)} - \frac{x(t)Q(t)}{L(t)} \quad (1.23)$$

$$\dot{q}(t) = B(\phi)(1 - z(t) - x(t))q(t) - z(t)q(t) - x(t)q(t) \quad (1.24)$$

1.1.5 Technology

The existing stock of technology, $A(t)$, and renewable natural capital devoted in the production of technological capital $x(t)Q(t)$(in the form of new ideas) alter the rate of technological advancement that hence increase technology accumulation. The equation of motion for technological capital is therefore given by:

$$\dot{A}(t) = \nu(\phi)(x(t)Q(t))^{\chi}(A(t)L(t))^{1-\chi} \tag{1.25}$$

$$\frac{\dot{A}(t)}{L(t)} = \frac{\nu(\phi)(x(t)Q(t))^{\chi}(A(t)L(t))^{1-\chi}}{L(t)} \tag{1.26}$$

$$\frac{\dot{A}(t)}{L(t)} = \nu(\phi)(x(t)q(t))^{\chi}A(t)^{1-\chi} \tag{1.27}$$

as we know that

$$a(t) = \frac{A(t)}{L(t)} \tag{1.28}$$

$$\dot{a}(t) = \frac{\dot{A}(t)}{L(t)} - \frac{A(t)}{L(t)^2}\dot{L}(t) \tag{1.29}$$

$$\dot{a}(t) = \frac{\dot{A}(t)}{L(t)} - a(t)\frac{\dot{L}(t)}{L(t)} \tag{1.30}$$

In this model, the population is constant therefore the population growth rate is normalized to one. $(L(t) = 1, \frac{\dot{L}(t)}{L(t)} = 0)$, so

$$\dot{a}(t) = \frac{\dot{A}(t)}{L(t)} = \dot{A}(t) \tag{1.31}$$

hence I can rewrite (1.27) as:

$$\dot{A}(t) = \nu(\phi)(x(t)q(t))^{\chi}A(t)^{1-\chi} \tag{1.32}$$

$$\frac{\dot{A}(t)}{A(t)} = \nu(\phi)\left(\frac{x(t)q(t)}{A(t)}\right)^{\chi} \tag{1.33}$$

Here $\nu(\phi)$ is the scale parameter that positively depends on the development of financial institutions (ϕ) as $\nu(\phi) > 1$. Sound financial systems increase the technology accumulation through effectively allocate savings and investments [Xu et al. (2014); Aghion et al. (2005); Aghion and Howitt (2008)]. Financial intermediation affect technology accumulation through two channels: $\nu(\phi)$ accounts for the direct channel and $\left(\frac{x(t)q(t)}{A(t)}\right)^{\chi}$ accounts for indirect channel through renewable natural capital devoted in the production of technological capital. χ is the share of technological capital, with $0 < \chi < 1$. This specification of technology accumulation is suggested in Jones (2005) and recently in La Torre and Marsiglio (2010).

1.2 The model

The model is expressed in per capita terms and can be written as:

$$y = A^{1-\alpha-\beta}k^{\alpha}(zq)^{\beta}, \tag{1.34}$$

$$\text{Max}_{c,z,x} \quad \int_0^{\infty} \frac{c^{1-\sigma}-1}{1-\sigma}e^{-\rho t}, \ \sigma \neq 1 \tag{1.35}$$

subject to the constraints on the evolution of physical capital and natural capital:

$$\dot{k} = [1-\xi(\phi)]A^{1-\alpha-\beta}k^{\alpha}(zq)^{\beta} - c - \delta k, \ k_0 = k(0), \tag{1.36}$$

$$\dot{q} = B(\phi)(1-z-x)q - zq - xq, \ Q_0 = Q(0), \tag{1.37}$$

$$\dot{A} = \nu(\phi)(xq)^{\chi}A^{1-\chi}, 0 < \chi < 1 \tag{1.38}$$

The variables c, q, k, A, z and x are the function of time t. The constrain for q is non-concave due to $(1 + B(\phi))qz$ and $(1 + B(\phi))qx$ terms, therefore Mangasarian's sufficiency theorem [see Mangasarian (1966)] cannot be employed. Arrow (1968) sufficiency theorem is utilized to check the sufficient conditions.

1.2.1 First-order conditions

The current value Hamiltonian for this model is

$$\begin{aligned} H = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda_1\Big[(1-\xi(\phi))k^{\alpha}(zq)^{\beta}A^{1-\alpha-\beta} - c - \delta k\Big] \\ +\lambda_2\Big[B(\phi)(1-z-x)q - zq - xq\Big] + \lambda_3\Big[\nu(\phi)(xq)^{\chi}A^{1-\chi}\Big], \end{aligned} \tag{1.39}$$

The first-order necessary conditions are [see Pontryagin (1987)]:

$$c: \qquad \lambda_1 = c^{-\sigma}, \tag{1.40}$$

$$z: \qquad \lambda_2(B(\phi)+1)q = \beta(1-\xi(\phi))\lambda_1 k^{\alpha} q^{\beta} A^{1-\alpha-\beta} z^{\beta-1}, \tag{1.41}$$

$$x: \qquad \nu(\phi)\lambda_3 \chi x^{\chi-1} q^{\chi} A^{1-\chi} = \lambda_2 q(B(\phi)+1), \tag{1.42}$$

$$k: \qquad \dot{\lambda}_1 = \lambda_1[-\alpha(1-\xi(\phi))k^{\alpha-1}(zq)^{\beta} A^{1-\alpha-\beta} + \rho + \delta], \tag{1.43}$$

$$q: \quad \dot{\lambda}_2 = -\beta(1-\xi(\phi))\lambda_1 k^{\alpha} z^{\beta} q^{\beta-1} A^{1-\alpha-\beta} - \lambda_3 \nu(\phi)\chi x^{\chi} q^{\chi-1} A^{1-\chi}$$
$$-\lambda_2[B(\phi)(1-z-x) - z - x - \rho], \tag{1.44}$$

$$A: \qquad \dot{\lambda}_3 = -\lambda_1(1-\alpha-\beta)(1-\xi(\phi))k^{\alpha}(zq)^{\beta} A^{-\alpha-\beta}$$
$$+\lambda_3[\rho - \nu(\phi)(1-\chi)(xq)^{\chi} A^{-\chi}], \tag{1.45}$$

$$\lambda_1: \qquad \dot{k} = (1-\xi(\phi))k^{\alpha}(zq)^{\beta} A^{1-\alpha-\beta} - c - \delta k, \tag{1.46}$$

$$\lambda_2: \qquad \dot{q} = B(\phi)(1-z-x)q - zq - xq, \tag{1.47}$$

$$\lambda_3: \qquad \dot{A} = \nu(\phi)(xq)^{\chi} A^{1-\chi}, \tag{1.48}$$

and the transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)k(t) = 0, \tag{1.49}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)q(t) = 0, \tag{1.50}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_3(t)A(t) = 0. \tag{1.51}$$

With the aid of equations (1.41) and (1.42) equations (1.43) and (1.44) can be re-written as

$$\dot{\lambda}_2 = \lambda_2(\rho - B(\phi)), \tag{1.52}$$

$$\dot{\lambda}_3 = \lambda_3\left[\rho - (1-\chi)\left(\frac{xq}{A}\right)^{\chi} - \frac{\chi(1-\alpha-\beta)}{\beta}\left(\frac{xq}{A}\right)^{\chi}\frac{z}{x}\right]. \tag{1.53}$$

Equations (1.40)- (1.42) yields following values of control variables:

$$c = \lambda_1^{-\frac{1}{\sigma}}, \tag{1.54}$$

$$z = \left(\frac{\lambda_2(B(\phi)+1)A^{\alpha+\beta-1}}{\beta(1-\xi(\phi))\lambda_1 k^{\alpha}}\right)^{\frac{1}{\beta-1}} \frac{1}{q}, \tag{1.55}$$

$$x = \left(\frac{\lambda_2(B(\phi)+1)}{\chi\nu(\phi)\lambda_3}\right)^{\frac{1}{\chi-1}} \frac{A}{q}. \tag{1.56}$$

The time derivatives of (1.54)- (1.56) yields following growth rates of control variables: The growth rates of the control variables (c, z, x) are

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}[\alpha(1-\xi(\phi))k^{\alpha-1}(zq)^{\beta}A^{1-\alpha-\beta} - \delta - \rho], \tag{1.57}$$

$$\begin{aligned}\frac{\dot{z}}{z} = \frac{1}{\beta-1}\Bigg[\alpha\frac{c}{k} - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)\left(\frac{xq}{A}\right)^{\chi} \\ -(1-\beta)(B(\phi)+1)(z+x)\Bigg],\end{aligned} \tag{1.58}$$

$$\begin{aligned}\frac{\dot{x}}{x} = \frac{1}{\chi-1}\Bigg[\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z}{x}\left(\frac{xq}{A}\right)^{\chi} - \chi B(\phi) \\ -(1-\chi)(B(\phi)+1)(z+x)\Bigg].\end{aligned} \tag{1.59}$$

1.2.2 Sufficiency conditions

The sufficiency of first-order conditions is established in the following Proposition by utilizing Arrow (1968) theorem:

Proposition 1:

The first-order conditions are sufficient as well.

Proof:

In order to check for sufficiency conditions, the values of control variables from (1.54)-(1.56) can be substituted in the current value Hamiltonian (1.39) to establish the maximized Hamiltonian. The maximized Hamiltonian is defined as

$$
\begin{aligned}
H^0(t,k,q,a,\lambda_1,\lambda_2,\lambda_3) &= \frac{\lambda^{\frac{\sigma-1}{\sigma}}-1}{1-\sigma} \\
+\lambda_1\Bigg[\left(\frac{\lambda_2(B(\phi)+1)}{\beta\lambda_1}\right)^{\frac{\beta(1-\xi(\phi))}{\beta-1}} & k^{\frac{\alpha}{1-\beta}}A^{\frac{1-\alpha-\beta}{1-\beta}} - \lambda_1^{-\frac{1}{\sigma}} - \delta k\Bigg] \\
+\lambda_2\Bigg[(B(\phi)+1)^{\frac{\beta}{\beta-1}}\left(\frac{\lambda_2}{\beta(1-\xi(\phi))\lambda_1}\right)^{\frac{1}{\beta-1}} & k^{\frac{\alpha}{1-\beta}}A^{\frac{1-\alpha-\beta}{1-\beta}} \\
-(B(\phi)+1)^{\frac{\chi}{\chi-1}}\left(\frac{\lambda_2}{\chi\nu(\phi)\lambda_3}\right)^{\frac{1}{\chi-1}} & A + qB(\phi)\Bigg] \\
+\lambda_3\left[\frac{\lambda_2(B(\phi)+1)}{\chi\nu(\phi)\lambda_3}\right]^{\frac{\chi}{\chi-1}} & A
\end{aligned} \tag{1.60}
$$

The maximized Hamiltonian (1.60) is always concave in state variables k, q and A as $\alpha + \beta < 1$. Therefore, I can conclude that the first-order conditions are sufficient by Arrow's theorem [see Arrow (1968)]. This completes proof of proposition 1.

1.3 Conclusion

The chapter presented the formulation of a three-sector finance extended endogenous growth model with renewable natural capital and physical capital. This chapters builds a theoretical framework by incorporating financial institutions in the form of financial intermediaries in all three sectors. Furthermore, the sufficiency conditions are checked through Arrow's sufficiency theorem. As Mangasarian's sufficiency theorem can not be employed because the constrain for q is non-concave due to $(1+B(\phi))qz$ and $(1+B(\phi))qx$ terms. The first-order conditions are proven to be sufficient by Arrow's theorem.

Chapter 2

The BGP equilibrium, functional forms and Numerical simulations

The chapter will analyze the BGP equilibrium of the model economy where all variables will grow at a constant rate. I will discuss the role of financial development on economic growth and BGP equilibrium by considering different functional forms of functions that depend on the degree of financial development such as the growth rate of renewable natural capital $B(\phi)$, scale parameter of technology $\nu(\phi)$ and the cost of financial intermediation $\xi(\phi)$. Later, the BGP equilibrium will be analyzed at the benchmark values of the key parameters. And the analysis will be in the context of developing and developed economies that how financial development lead to differences in country's accumulation of capital and economic growth.

2.1 Steady state solution

A balanced growth path is a sequence of time path along which all economic variables (c, q, k, A, z, x) grow at a constant rate. For that, I will solve the model for steady state values. The solution does not exist for the original variables so I have used dimensionality reduction technique by taking ratios of variables. I have used this technique by following La Torre and Marsiglio (2010) approach. Therefore I obtained the following system of five nonlinear differential equations by introducing the variables: $U = \frac{c}{k}$, $V = \frac{q}{k}$ and $W = \frac{A}{k}$ (see appendix A):

$$\frac{\dot{U}}{U} = \frac{1}{\sigma}\left[(\alpha - \sigma)(1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} - (1-\sigma)\delta - \rho + \sigma U\right] \tag{2.1}$$

$$\frac{\dot{V}}{V} = B(\phi)(1 - z - x) - z - x - (1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} + \delta + U \tag{2.2}$$

$$\frac{\dot{W}}{W} = \nu(\phi)\left(\frac{xV}{W}\right)^{\chi} - (1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} + \delta + U \tag{2.3}$$

$$\begin{aligned}\frac{\dot{z}}{z} = \frac{1}{\beta - 1}\Bigg[&\alpha U - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)\left(\frac{xV}{W}\right)^{\chi} \\ &-(1-\beta)(B(\phi)+1)(z+x)\Bigg],\end{aligned} \tag{2.4}$$

$$\begin{aligned}\frac{\dot{x}}{x} = \frac{1}{\chi - 1}\Bigg[&\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z}{x}\left(\frac{xV}{W}\right)^{\chi} - \chi B(\phi) \\ &-(1-\chi)(B(\phi)+1)(z+x)\Bigg].\end{aligned} \tag{2.5}$$

Furthermore by introducing the variables $R = (zV)^{\beta}W^{1-\alpha-\beta}$ and $S = \left(\frac{xV}{W}\right)^{\chi}$, I can rewrite the system as follows:

$$\frac{\dot{U}}{U} = \frac{1}{\sigma}\left[(\alpha - \sigma)(1 - \xi(\phi))R - (1-\sigma)\delta - \rho + \sigma U\right] \tag{2.6}$$

$$\begin{aligned}\frac{\dot{R}}{R} = &\left(\frac{\alpha+\beta-1}{\beta-1}\right)U - \left(\frac{\beta}{\beta-1}\right)B(\phi) - (1-\alpha)(1-\xi(\phi))R \\ &-\left(\frac{1-\alpha-\beta}{\beta-1}\right)\nu(\phi)S - \left(\frac{1-\alpha}{\beta-1}\right)\delta\end{aligned} \tag{2.7}$$

$$\frac{\dot{S}}{S} = \frac{\chi^2\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}\frac{z}{x}S - \chi\nu(\phi)S - \left(\frac{\chi}{\chi-1}\right)B(\phi) \tag{2.8}$$

$$\begin{aligned}\frac{\dot{z}}{z} = \frac{1}{\beta-1}\Big[&\alpha U - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)S \\ &-(1-\beta)(B(\phi)+1)(z+x)\Big]\end{aligned} \tag{2.9}$$

$$\begin{aligned}\frac{\dot{x}}{x} = \frac{1}{\chi-1}\Big[&\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z}{x}S - \chi B(\phi) \\ &-(1-\chi)(B(\phi)+1)(z+x)\Big]\end{aligned} \tag{2.10}$$

The existence and non-negativeness of the steady state values is established in the following propositions.

Proposition 2:

The steady state solution exist for all set of variables $(U^*, R^*, S^*, z^*, x^*)$.

Proof:

The steady state equilibrium point is where equations (2.6)-(2.10) are equal to zero (See Appendix B for steady state solution). The steady state values $(U^*, R^*, S^*, z^*, x^*)$ are given by:

$$U^* = \frac{B(\phi)\sigma - \alpha(B(\phi)-\rho)}{\alpha\sigma} + \frac{(1-\alpha)}{\alpha}\delta \tag{2.11}$$

$$R^* = \frac{B(\phi)+\delta}{\alpha(1-\xi(\phi))} \tag{2.12}$$

$$S^* = \frac{B(\phi)-\rho}{\sigma\nu(\phi)} \tag{2.13}$$

$$z^* = \frac{(\sigma B(\phi)+\rho-B(\phi))[\beta\chi(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]}{\sigma(B(\phi)+1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]} \tag{2.14}$$

$$x^* = \frac{\chi(1-\alpha-\beta)(B(\phi)-\rho)(\sigma B(\phi)+\rho-B(\phi))}{\sigma(B(\phi)+1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]} \tag{2.15}$$

by using the values of steady state I can also derive value for V^* and W^*:

$$V^* = \left(\frac{{x^*}^{\alpha+\beta-1}R^*}{{z^*}^{\beta}{S^*}^{\frac{\alpha+\beta-1}{\chi}}}\right)^{\frac{1}{1-\alpha}} \tag{2.16}$$

$$W^* = \left(\frac{x^* R^{*\frac{1}{\beta}}}{z^* S^{*\frac{1}{\chi}}} \right)^{\frac{\beta}{1-\alpha}} \tag{2.17}$$

The steady state equilibrium values are verified by using MAPLE 18 as well. This completes the proof of Proposition 2.

Proposition 3:

The balance growth path is characterized by a strict positive level of consumption, physical capital, natural capital, technological capital, shares of natural capital allocated to the production of final good and technology sectors provided the following parameter restrictions hold:

$$B(\phi)(1-\sigma) < \rho < B(\phi), \tag{2.18}$$

$$\sigma > \frac{(B(\phi)-\rho)\alpha}{B(\phi)+(1-\alpha)\delta}, \tag{2.19}$$

$$B(\phi)+\delta > 0, \tag{2.20}$$

$$B(\phi)+\delta-\alpha\delta > 0. \tag{2.21}$$

Proof:

I will prove the non-negativeness of steady state values $U^*, V^*, W^*, R^*, S^*, z^*, x^*$. Note that $x^* > 0$ when $B(\phi) - \rho > 0$ and $\rho - (1-\sigma)B(\phi) > 0$ which yields

$$B(\phi)(1-\sigma) < \rho < B(\phi) \tag{2.22}$$

This guarantees z^* and S^* are positive as well. The variables V^* and W^* are positive provided that R^* is positive. Thus I requires

$$B(\phi)+\delta > 0 \tag{2.23}$$

Lastly, the positiveness of U^* is guaranteed provided

$$\sigma > \frac{(B(\phi)-\rho)\alpha}{B(\phi)+(1-\alpha)\delta} \tag{2.24}$$

$$B(\phi)+\delta-\alpha\delta > 0 \tag{2.25}$$

This completes the proof of Proposition 3.

Proposition 4: There exists a common growth rate in the economy. The variables k, q, A, c have a common growth rate denoted by g.

Proof:

The common growth rate of variables c, k, q, A can be computed with the aid of (1.46)-(1.48) and (1.57):

$$\frac{\dot{c}}{c} = \frac{\alpha(1-\xi(\phi))R - \delta - \rho}{\sigma} \tag{2.26}$$

$$\frac{\dot{k}}{k} = (1-\xi(\phi))R - U - \delta \tag{2.27}$$

$$\frac{\dot{q}}{q} = B(\phi) - z(B(\phi)+1) - x(B(\phi)+1) \tag{2.28}$$

$$\frac{\dot{A}}{A} = \nu(\phi)S \tag{2.29}$$

by substituting the value of U^*, R^*, S^*, z^*, x^* from equations (2.11)-(2.15), the common BGP growth rate can be written as:

$$g = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{q}}{q} = \frac{\dot{A}}{A} = \frac{B(\phi) - \rho}{\sigma} > 0 \tag{2.30}$$

This completes the proof of Proposition 4. The growth rate is strictly positive provided $\rho < B(\phi)$ holds true. It is clear from equation (2.30) that economic growth positively depends on the growth rate of renewable natural capital, $B(\phi)$. Therefore, the engine of economic growth is ultimately dependent on the investment in the renewable natural capital through financial development. The degree by which financial development impacts the growth rate of renewable natural capital is very crucial for determining the economic growth. The effect of financial development on economic growth can be examined as follows:

$$\frac{\partial g}{\partial \phi} = \frac{B'(\phi)}{\sigma} \tag{2.31}$$

In equation (2.31), the term $\frac{B'(\phi)}{\sigma}$ determines the impact of financial development on economic growth, which I refer as productivity effect (similar to the productivity effect of human capital discussed by Bucci and Marsiglio (2018)). The well developed financial sector increases the productivity of investment in renewable natural capital in terms of high growth of renewable natural capital stock. This positive relationship between financial development and economic growth holds true when $B'(\phi) > 0$.

2.2 The functional forms

In finance growth nexus it is evident that financial development is crucial for determining economic growth through the channel of investment [Bencivenga and Smith (1991); Xu (2000); Carlin and Mayer (2003)]. In my model, financial development affect the BGP equilibrium through three functions: the growth rate of renewable natural capital ($B(\phi)$), scale parameter of technology ($\nu(\phi)$) and the cost of financial intermediation ($\xi(\phi)$).

The effect of financial development on economic growth and steady state equilibrium is stronger in early stages of development [Rioja and Valev (2004); Fung (2009)]. And afterwards financial development has a vanishing effect [Rousseau and Wachtel (2011); Arcand et al. (2015)]. This relationship also feed into the debate of great divergence, Fung (2009) found that countries with relatively developed financial institutions have a tendency to grow faster and converge to a higher BGP equilibrium. Therefore, developing and emerging economies with well developed financial sectors grow faster and catch up with developed economies. In this regard, I have taken different functional forms of $B(\phi)$, $\nu(\phi)$ and $\xi(\phi)$ with respect to developed and developing economies. The classification of these economies is according to the global frontier.

2.2.1 The functional form of $B(\phi)$

The objective of this thesis is to unwind the puzzle of economic growth by building a three sector endogenous growth model by incorporating financial institutions and endogenous technology. In renewable natural resource sector financial institutions play an important role as financial development positively contribute to the accumulation of renewable natural capital through easing the financing constraints (higher ϕ leads to a higher $B(\phi)$) [Brunnschweiler (2010); Kim and Park (2016)]. Furthermore from equation (2.31), it is evident that financial development positively contribute to economic growth through productivity effect of renewable natural capital. This positive relationship between financial development and economic growth holds true when $B'(\phi) > 0$. In this regard, I have taken different functional forms of $B(\phi)$ with respect to developing and developed economies.

Case 2.2.1.1 Developed Economies

I will consider $B(\phi)$ as square root function[1] of ϕ for developed economies:

$$B(\phi) = a_1\sqrt{\phi} \tag{2.32}$$

In this case, it is clearly visible that $B'(\phi) > 0$ provided $a_1 > 0$ which concludes that the financial development and economic growth has positive and monotonic relationship. However, in developed economies the effect of financial development on $B(\phi)$ diminishes as illustrated in Figure (2.1) (by considering $a_1 = 0.35$).

Case 2.2.1.2 Developing Economies

I will consider simple linear form in which $B(\phi)$ is a linear function of ϕ for developing economies:

$$B(\phi) = a_1\phi \tag{2.33}$$

here, $B'(\phi)$ is also greater than zero (provided $a_1 > 0$) and reinforces strictly positive and monotonic relationship of financial development and economic growth (illustrated in Figure (2.1))

[1] Jeanblanc et al. (2009) used radical/square root functional forms in analyzing financial markets.

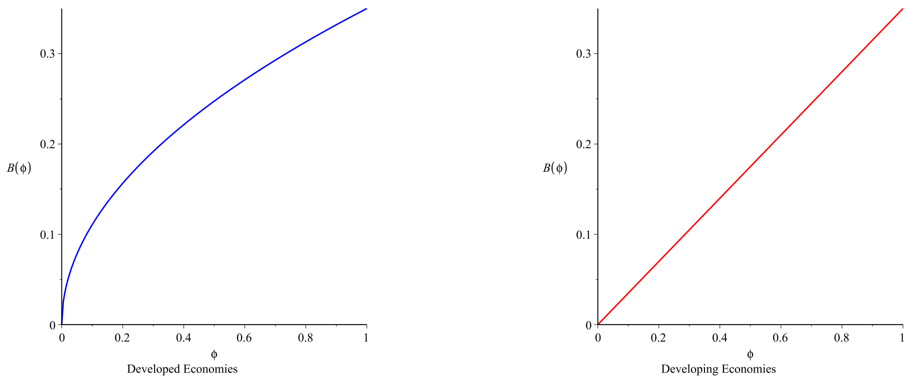

Figure 2.1: Effect of financial development on $B(\phi)$ in Developed vs Developing Economies

The findings clearly state the positive and monotonic relationship exists between financial development and economic growth through productive investments in renewable natural capital. This finding is also supported by empirical literature where it is evident that the financial sector's depth and composition are the key determinant of mobilizing private investment in renewable energy projects [Saidi (2006); Ba et al. (2010); Calitz and Fourie (2010); Mathews et al. (2010); Scholtens and Veldhuis (2015)]. Therefore, the well-developed financial sector have positive relationship with renewable energy produced in an economy and hence contribute to economic growth [Ba et al. (2010); Brunnschweiler (2010); Marques and Fuinhas (2011); Scholtens and Veldhuis (2015)].

After the financial crisis of 2008-09, some empirical papers found the negative impact of financial development on economic growth after reaching a threshold level of financial deepening [Arcand et al. (2015); Aizenman et al. (2015); Samargandi (2015); Soedarmono et al. (2017); Bucci and Marsiglio (2018)]. All of these are consistent with Rousseau and Wachtel (2011)'s "vanishing effect". As Bucci and Marsiglio (2018) found non-monotonic relationship between finance and economic growth through human capital accumulation where the depreciation effect off sets the productivity effect. Whereas in this regard it is important to understand the debate raised by Rajan and Zingales (1998) that financial development facilitate economic growth in those industries that heavily rely on external financing. Intriguingly Arcand et al. (2015) found that the effect of finance on output growth varies across sectors.

The main finding from my model so far suggests that in long run (BGP) financial development is positively and monotonically related to economic growth through the productivity effect of renewable natural capital investments. Renewable natural capital accumulation heavily rely on external financing so increasing financial deepening not only ease the credit channel but also enhance the effectiveness of credit.

2.2.2 The functional form of $\nu(\phi)$

Financial development directly impact the rate of technological advancement by a scale parameter $\nu(\phi)$. A well developed financial sector direct resources towards more R&D intensive sectors where growth is driven by technology, as a result countries with well developed financial sectors display higher output growth through greater R&D intensity [Aghion et al. (2005), Aghion and Howitt (2008); Agn (2011); Ilyina and Samaniego (2011)]. Moreover, the effect of financial development on technological advancement is more pronounced in developing economies and as it reaches to the global frontier, the effect of financial development of economic growth diminishes [Berthelemy and Varoudakis (1996); Aghion et al. (2005); Xu et al. (2014)]. I will study the role of financial development by considering different functional forms of $\nu(\phi)$ with respect to developed and developing economies.

Case 2.2.2.1 Developed Economies

Financial development positively related to the technological advancement. In developed economies, the rate at which financial development effect scale parameter of technology ($\nu(\phi)$) diminishes as countries converge to global growth frontier. Therefore I have considered following functional forms of $\nu(\phi)$:

$$\nu(\phi) = a_2\sqrt{\phi} \tag{2.34}$$

with $a_2 > 0$ such that $\nu'(\phi) > 0$ and $\nu''(\phi) < 0$ as illustrated through graphs in Figure (2.2) (considering $a_2 = 0.5$).

Case 2.2.2.2 Developing Economies

The effect of financial development on technological advancement is higher in developing economies [Aghion et al. (2005); Aghion and Howitt (2008); Xu et al. (2014)]. Therefore, I have considered following functional forms of $\nu(\phi)$ with respect to ϕ;

$$\nu(\phi) = a_2\phi \tag{2.35}$$

such that $\nu'(\phi) > 0$ as illustrated through graphs in Figure (2.2).

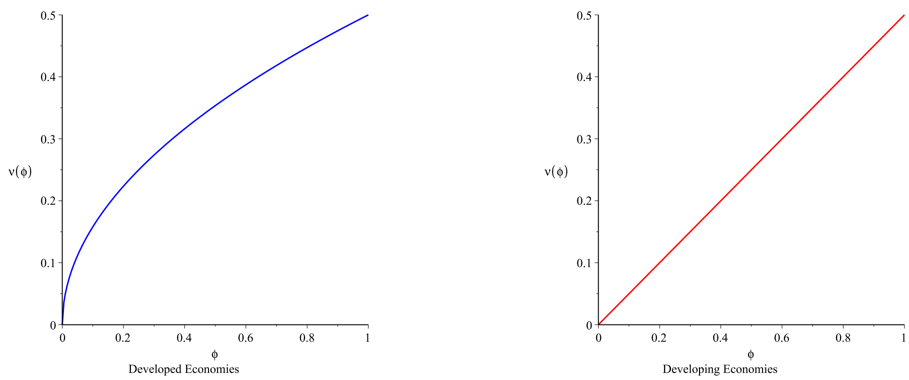

Figure 2.2: Effect of financial development on $\nu(\phi)$ in Developed vs Developing Economies

2.2.3 The functional form of $\xi(\phi)$

Financial intermediaries charge fee for the supply of services, it is subtracted from the total output that otherwise contribute to the investment of physical capital. Empirically, the cost of financial intermediation ($\xi(\phi)$) decreases with the development of financial sector up to a threshold level and after that it become constant [Philippon (2015)]. The decrease in the financial intermediation cost is higher in the early stages of development and after a certain threshold level it become constant for both developing and developed economies. Therefore, the effect of financial development on economic growth through capital accumulation is stronger in developing economies [Rioja and Valev (2004); Fung (2009)]. I will study the role of financial development by considering different functional forms of $\xi(\phi)$ with respect to developed and developing economies.

Case 2.2.3.1 Developed Economies

Financial development negatively affects the financial intermediation cost. In developed economies, the rate at which financial development effect the cost of financial intermediation ($\xi(\phi)$) diminishes as countries converge to global growth frontier. Therefore I have considered following functional forms of $\xi(\phi)$:

$$\xi(\phi) = \frac{a_3}{a_3 + \sqrt{\phi}} \tag{2.36}$$

with $a_3 > 0$ such that $\xi'(\phi) < 0$ and $\xi''(\phi) > 0$ as illustrated through graphs in Figure (2.3) (considering $a_3 = 0.025$).

Case 2.2.3.2 Developing Economies

In developing economies, financial intermediation cost $\xi(\phi)$ decreases at a faster rate with the improvement in financial development (higher ϕ) [Rioja and Valev (2004); Philippon (2015)]. Therefore, I have considered following functional forms of $\xi(\phi)$ with respect to ϕ;

$$\xi(\phi) = \frac{a_3}{a_3 + \phi} \tag{2.37}$$

with $a_3 > 0$ such that $\xi'(\phi) < 0$ and $\xi''(\phi) > 0$ as illustrated through graphs in Figure (2.3).

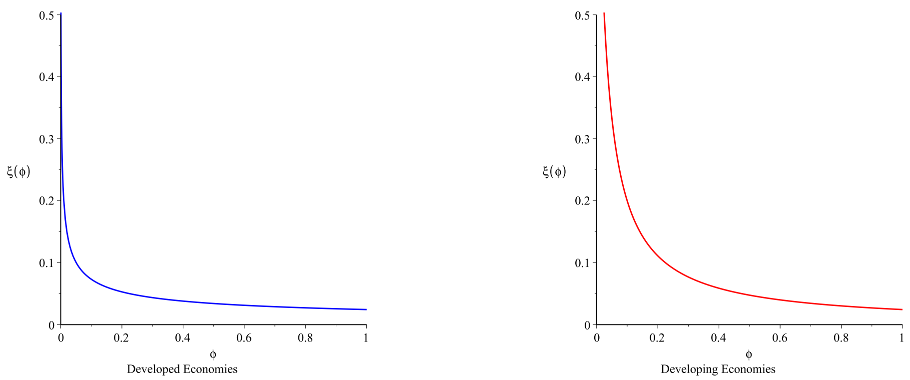

Figure 2.3: Effect of financial development on $\xi(\phi)$ in Developed vs Developing Economies

2.3 An analysis of BGP equilibrium at benchmark values

Financial development plays a central role in determining economic growth and BGP equilibrium. In order to analyze the BGP equilibrium I have consider the benchmark values for the key parameters as follows in Table (2.1) [La Torre and Marsiglio (2010); Chaudhry et al. (2017); Bucci and Marsiglio (2018)].

The value of the share of physical capital (α) is assumed around one third in Mankiw et al. (1992) and recently in La Torre and Marsiglio (2010). Hence I have considered the share of physical capital as 0.33. Likewise, according to Kortum (1993) the share of idea in technology production varies between 0.1 to 0.6, hence I have assumed $\chi = 0.42$. By following Mulligan and Sala-i-Martin (1993) and Antoci et al. (2011), I have set $\rho = 0.04$ and $\sigma = 1.5$ respectively.

Table 2.1: Parameters-Benchmark Values

α	β	χ	σ	ϕ	ρ	δ	a_1	a_2	a_3
0.33	0.31	0.42	1.5	0.32	0.04	0.1	0.35	0.5	0.025

The lower-bound value for ϕ can be determined by the condition that ensures non-negativeness of the BGP equilibrium ($B(\phi)(1-\sigma) < \rho < B(\phi)$). As, I have considered the different functional forms for developing and developed economies therefore the bounds of ϕ will vary for both. The lower bound value of ϕ for developing and developed economies are $\phi > 0.1143$ and $\phi > 0.0131$ respectively. However, the upper bound of ϕ can be computed by the constraints of the rate of aggregate extraction of renewable natural capital ($x + z < 1$). The upper bound value is $\phi < \infty$. Hence the benchmark value of $\phi = 0.32$ will provide strictly positive BGP equilibrium values.

The functional forms of $B(\phi), \nu(\phi)$ and $\xi(\phi)$ through which financial development affects the BGP equilibrium are crucial for studying finance growth nexus, hence $B(\phi), \nu(\phi)$ and $\xi(\phi)$ at benchmark values are in Table (2.2).

Table 2.2: $B(\phi), \nu(\phi)$ and $\xi(\phi)$ for Developed and Developing Economies at Benchmark Values

Type of Economy	$B(\phi)$	$\nu(\phi)$	$\xi(\phi)$
Developed Economies	0.1979	0.2828	0.0423
Developing Economies	0.1120	0.1600	0.0724

Higher cost of financial intermediation is negatively associated with income levels because of low level of financial development [Calice and Zhou (2018)]. Hence it is evident in Table (2.2) that the cost of financial intermediation ($\xi(\phi)$) is high in developing economies with respect to developed economies. The cost of financial intermediation is further associated with credit rationing therefore when $\xi(\phi)$ is high, low level of credit is channeled in investors to accumulate capital [Calice and Zhou (2018)]. As a result the growth rate of renewable natural capital ($B(\phi)$) and scale parameter of technology ($\nu(\phi)$) is also low in developing economies with respect to developed economies.

By using the values of $B(\phi)$, $\nu(\phi)$ and $\xi(\phi)$ I calculated BGP equilibrium values for developing and developed economies at $(\phi) = 0.32$ (Table (2.3)). It is evident that the steady state equilibrium values $R^* = y/k$, $S^* = \left(\frac{xq}{A}\right)^{\chi}$, $U^* = c/k$, $V^* = q/k$, $W^* = A/k$, extraction rate used in the production of final consumption good (z^*) and used in the production of technological capital (x^*) are higher in developed economies with respect to developing economies. The aggregate extraction rates of renewable natural capital used in the production of final consumption good (z^*) and technological capital (x^*) are positively dependent on the growth rate of renewable natural capital $B(\phi)$. At $\phi = 0.32$, developed economies have 8.6 per cent higher growth rate of renewable natural capital (Table (2.2)) compared to developing economies therefore the steady state values of extraction rates used in the production of final consumption good (z^*) and technological capital (x^*) are higher in developed economies.

Table 2.3: BGP equilibrium at Benchmark Values

Type of Economy	R^*	S^*	U^*	V^*	W^*	z^*	x^*
Developed	0.942	0.372	0.697	7.793	1.728	0.056	0.021
Developing	0.692	0.300	0.494	5.471	1.204	0.045	0.013

The steady state equilibrium values $R^* = y/k$, $S^* = \left(\frac{xq}{A}\right)^{\chi}$, $U^* = c/k$, $V^* = q/k$, $W^* = A/k$ are positively depend on growth rate of renewable natural capital ($B(\phi)$) and scale parameter of technology ($\nu(\phi)$) while negatively related to the cost of financial intermediation ($\xi(\phi)$). And it is evident from Table (2.2) that developed economies have low cost of financial intermediation in addition to high growth rate of renewable natural capital and scale parameter of technology. Therefore the steady state equilibrium values are higher in developed economies with respect to developing. At a given level of financial development ($\phi = 0.32$), stock of renewable natural capital per capita (q), stock of technology (A) and consumption per capita (c) is higher for developed economies and as a result the output per capita (y) is also higher.

2.4 Conclusion

In this chapter firstly I have proven that the unique steady state solution and common growth rate exist for my model. Secondly I determine that financial development impacts economic growth and BGP equilibrium through three functions, growth rate of renewable natural capital ($B(\phi)$), scale parameter of technology ($\nu(\phi)$) and the cost of financial intermediation ($\xi(\phi)$), and the functional forms of these functions varies across developing and developed economies. Lastly I analyzed the BGP equilibrium at benchmark values where and found that at $\phi = 0.32$, the steady states values are higher for developed economies with respect to developing.

Chapter 3

The Stability analysis and Numerical simulations

A stability analysis will be performed in this chapter. Firstly I will check stability of the model by building Jacobian matrix which will be evaluated at the steady state equilibrium. Secondly, I will perform numerical simulations where I will access the stability of the BGP at benchmark values for both developed and developing economies.

3.1 Stability analysis

The stability of the BGP equilibrium is checked by building Jacobian matrix of dynamic system (equations (2.6)-(2.10)) which is then evaluated at the steady state values. If all eigenvalues are strictly negative then the BGP equilibrium is asymptotically stable, whereas if at least one eigenvalue is negative then BGP equilibrium is saddle point stable, otherwise it is unstable.

In order to study the local stability of the system $(U^*, R^*, S^*, z^*, x^*)$ I will evaluate the trace and determinant of Jacobian matrix at the steady state values. As, the stability of the model is dependent on the sign of the trace and determinant. The trace and determinant determine eigenvalues such that trace is the sum of eigenvalues however determinant is the product of eigenvalues:

$$tr(J) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5, \tag{3.1}$$

$$det(J) = \lambda_1\lambda_2\lambda_3\lambda_4\lambda_5 \tag{3.2}$$

The trace and determinant of $J^*(U^*, R^*, S^*, z^*, x^*)$ are calculated by using MAPLE 18 and given by (See Appendix C):

$$tr(J^*(U^*, R^*, S^*, z^*, x^*)) = \frac{3(\rho + (\sigma - 1)B(\phi))}{\sigma} \tag{3.3}$$

$$\begin{aligned} det(J^*(U^*, R^*, S^*, z^*, x^*)) = \frac{1}{(\sigma^4(1-\chi)\alpha(1-\beta))}((\sigma - 1)B(\phi) + \rho) \\ \times(B(\phi) - \rho)((-\alpha + \sigma)B(\phi) - (-1 + \alpha)\delta\sigma + \alpha\rho)(B(\phi) + \delta)\chi \\ \times(-1 + \alpha)((-\chi - \sigma + 1)B(\phi) + \rho(\chi - 1)) \end{aligned}$$

which are positive provided equation (2.22) - (2.25) holds true. The same condition also ensures the non-negativeness of the steady state values of variables $(U^*, R^*, S^*, z^*, x^*)$. The determinant and trace of Jacobian matrix at the steady state values are positive therefore, if the eigenvalues are real numbers the possibilities of local stability are given by:

(1) provided $tr(J) > 0$ and $det(J) > 0$, the number of negative eigenvalues are either two or four (and one or three positive eigenvalues) then BGP equilibrium is saddle path stable.

(2) provided $tr(J) > 0$ and $det(J) > 0$, the number of negative eigenvalues are zero (and five positive eigenvalues) then BGP equilibrium is unstable.

The BGP equilibrium is either saddle path stable of dimension two or four, or it will be unstable. Therefore, it is critical to check the stability at benchmark values. I will perform numerical simulations to access the stability of the BGP for both developed and developing economies in next section.

3.2 Numerical simulations

I will conduct stability analysis by numerical examples. I have considered two numerical examples by taking functional forms of functions that depend of financial development such as $B(\phi), \nu(\phi)$ and the cost of financial intermediation ($\xi(\phi)$). I have considered linear and radical functional forms for developing and developed economies respectively. By using standard parameter values from Table (2.1), I have computed Jacobian matrix at steady state values and further computed eigenvalues and eigenvectors for both examples.

There are three possibilities for local stability:

(1) if all eigenvalues are strictly negative then BGP equilibrium will be stable,

(2) if at least one eigenvalue is negative then BGP equilibrium will be saddle path stable and it's dimension depends on number of negative eigenvalues and lastly,

(3) if all eigenvalues are strictly positive then BGP equilibrium will be unstable.

As in my model, the trace and determinant is positive so BGP equilibrium is either saddle path stable or unstable.

3.2.1 Stability analysis at benchmark values for Developed Economies

I have considered radical functional forms for developed economies (refer equations (2.32), (2.34) and (2.36)). And I have used standard parameters values from Table (2.1) to compute Jacobian matrix at steady state values. Further eigenvalues and eigenvectors are computed by using MAPLE 18.

Table 3.1: Eigenvalues and Eigenvectors at Parameters Benchmark Values (Developed Economies)

$$\lambda_1 = 0.0926$$
$$v_1 = [0, 0, 0, 1.5152, 0.5685]$$

$$\lambda_2 = 0.1710$$
$$v_2 = [-2.3556, -2.3805, -4.9480, -0.4802, -1.0695]$$

$$\lambda_3 = 0.4456$$
$$v_3 = [1.4481, 0.7002, 0.1678, -0.0956, 0.0206]$$

$$\lambda_4 = -0.0783$$
$$v_4 = [0.5068, 0.7547, 1.0646, 0.0124, 0.0442]$$

$$\lambda_5 = -0.3529$$
$$v_5 = [0.4436, 0.8943, 0.05142, 0.0267, 0.0038]$$

Developed economies show saddle path stable transitional manifold of dimension two as there are two negative eigenvalues. Hence the developed economy will converge to steady state equilibrium through a saddle path.

3.2.2 Stability analysis at benchmark values for Developing Economies

I have considered linear functional forms for developing economies (refer equations (2.33), (2.35) and (2.37)). And I have used standard parameters values from Table (2.1) to compute Jacobian matrix at steady state values. Further eigenvalues and eigenvectors are computed by using MAPLE 18.

Table 3.2: Eigenvalues and Eigenvectors at Parameters Benchmark Values (Developing Economies)

$$\lambda_1 = 0.0639$$
$$v_1 = [0, 0, 0, 1.9805, 0.5509]$$

$$\lambda_2 = 0.1004$$
$$v_2 = [-1.4512, -1.5983, -5.6056, -0.6783, -0.8859]$$

$$\lambda_3 = 0.3188$$
$$v_3 = [1.4245, 0.6990, 0.1559, -0.1096, 0.0122]$$

$$\lambda_4 = -0.0364$$
$$v_4 = [0.2678, 0.3975, 1.0348, 0.0029, 0.0324]$$

$$\lambda_5 = -0.2548$$
$$v_5 = [0.4301, 0.9010, 0.0471, 0.0293, 0.0025]$$

Developing economies show saddle path stable transitional manifold of dimension two as there are two negative eigenvalues. Hence developing economy will converge to steady state equilibrium through a saddle path.

3.3 Conclusion

This chapter has analyzed the stability of the BGP equilibrium. I built Jacobian matrix at steady state values which is firstly analyzed by the trace and determinant of the matrix. Later, stability analysis is performed by numerical simulations where I used functional forms and benchmark values from chapter 2. Numerical simulations are performed separately for developed and developing economies. And for both economies, the BGP equilibrium shows saddle path stable transitional manifold of dimension two. Therefore the developed as well developing economies will converge to steady state equilibrium through a saddle path.

Chapter 4

Effect of financial development on BGP equilibrium and Numerical simulations

In this chapter, I will analyze how financial development affects BGP equilibrium and the analysis is based on two comparison groups, developing and developed economies. In the later sections, I will perform numerical simulations by analyzing the effect of change of different parameters (share of physical capital, share of natural capital, share of technological capital and inverse of intertemporal elasticity of substitution) on BGP equilibrium and this is also in comparison between developing and developed economies.

4.1 Effect of financial development on BGP equilibrium

It is clear from Chapter 2 that financial development affects BGP equilibrium through growth rate of renewable natural capital ($B(\phi)$), scale parameter of technology ($\nu(\phi)$) and the cost of financial intermediation ($\xi(\phi)$). However it is critical to analyze the effect of change of financial development on BGP equilibrium. The effect of financial development on BGP equilibrium is analyzed through graphical illustration (Figure (4.1) - (4.2)).

The effect of financial development on the aggregate extraction rates of renewable natural capital used in the production of final consumption good (z^*) and technological capital (x^*) are positive for both developing and developed economies (clearly visible in the Figure (4.1)). The effect of financial development on capital accumulation is higher in early stages of development so it is more pronounced in developing economies [Rioja and Valev (2004); Fung (2009)]. As countries sustain their economic growth financial development tends to have a vanishing effect (in developed economies the effect of financial development on growth rate of renewable natural capital ($B(\phi)$) diminishes) [Rousseau and Wachtel (2011)]. Therefore with the increase in financial development, the steady state values of extraction rates used in the production of final consumption good (z^*) and technological capital (x^*) for developing economies tends to increase at a faster rate due to high growth rate of renewable natural capital (strictly positive relationship exist between $B(\phi)$ and ϕ). Hence I can conclude that strong financial institutions in developing countries increase the growth rate of renewable natural capital with simultaneous increase in the usage for the production of final consumption good and technological capital, that in result increase the speed of convergence for developing economies [Ilyina and Samaniego (2011)].

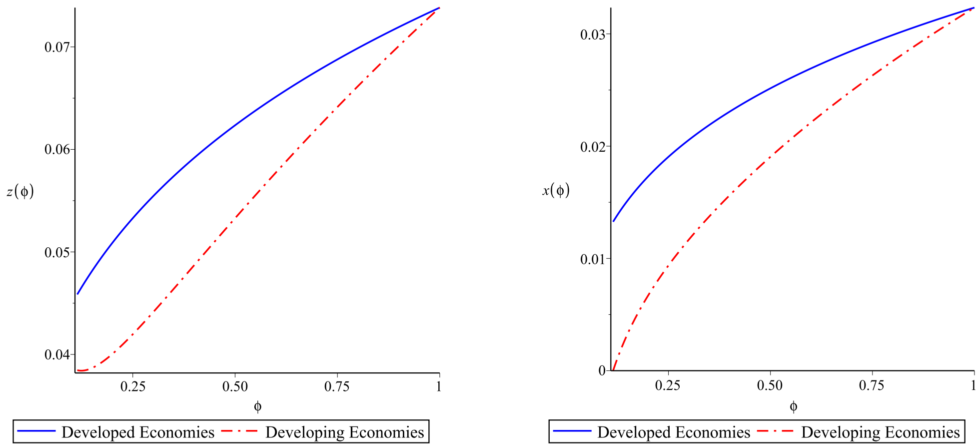

Figure 4.1: Effect of financial development on z^* and x^* in Developed vs Developing Economies

The effect of financial development on other key variables $W^* = A/k$, $V^* = q/k$, $U^* = c/k$ and $R^* = y/k$ is visible in Figure (4.2). The effect of financial development on y, c, q and A are positive i.e. monotonic for both developing and developed economies whereas in the case of developed economies financial development has vanishing effect as suggested by Rousseau and Wachtel (2011) and later by Arcand et al. (2015). Developing economies have insufficient amount of investment in order to raise capital, therefore development in financial sector increases the quantity and quality of R&D investment by lowering the cost of financial intermediation ($\xi(\phi)$) and increasing the scale parameter of technology ($\nu(\phi)$). Hence financial development increases the stock of technology A^* by effectively allocating resources [Xu et al. (2014)] and as a result increase the ratio of technology to physical capital ($W^* = A/k$). However the decrease in the financial intermediation cost is higher in the early stages of development [Philippon (2015)], therefore financial development contributes to innovation to a greater extent in developing economies than in developed ones [Aghion et al. (2005); Aghion and Howitt (2008); Xu et al. (2014)].

Moreover, in building the stock of renewable natural capital two forces play a critical role: firstly the financing for renewable energy projects and secondly the stock of technology available, and both of these are dependent on the development of financial sector [Ba et al. (2010); Calitz and Fourie (2010); Mathews et al. (2010); Scholtens and Veldhuis (2015)]. The financial sector's depth and composition are the key determinant of mobilizing private investment in renewable energy projects and hence increases the per capita stock of renewable natural capital (q^*) and as a result increases the ratio of renewable natural capital to physical capital ($V^* = q/k$) [Ba et al. (2010); Brunnschweiler (2010); Marques and Fuinhas (2011); Scholtens and Veldhuis (2015)]. While the investment constraints in renewable natural resource sector is higher for developing economies in comparison with developed economies and requires greater financial deepening [Lucon et al. (2006)]. Therefore at low ϕ, the ratio of renewable natural capital to physical capital (V^*) is higher for developed economies but as financial sector develops the ratio of renewable natural capital to physical capital (V^*) increases at a faster rate for developing economies.

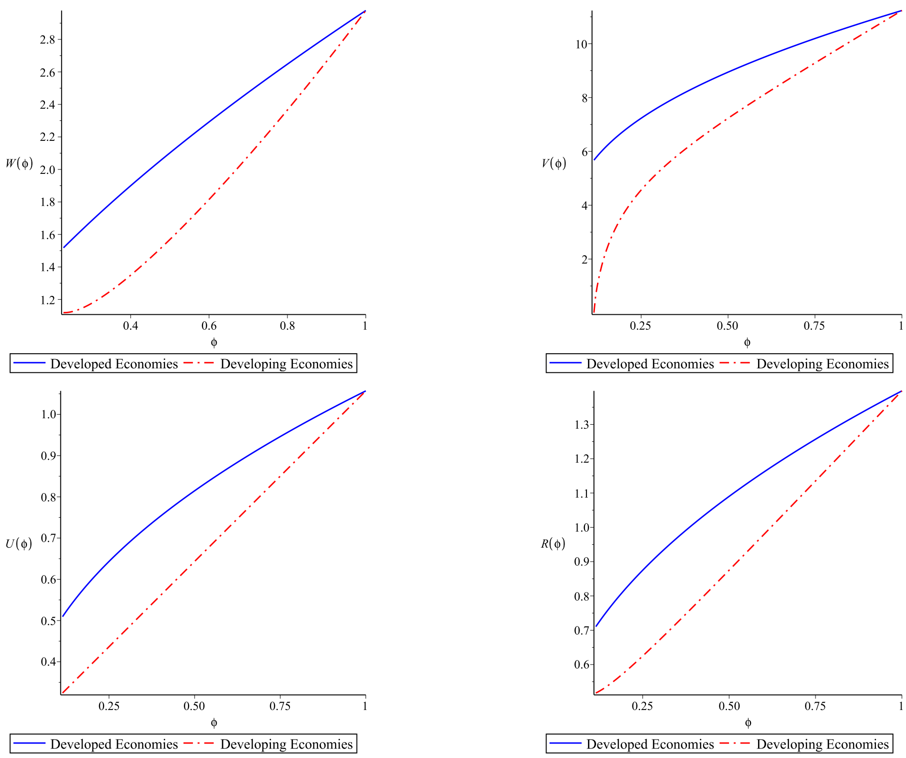

Figure 4.2: Effect of financial development on W^*, V^*, U^* and R^* in Developed vs Developing Economies

The increase in extraction rate used in the production of final consumption good (z^*), extraction rate used in the production of technological capital (x^*), the ratio of technological capital to physical capital (W^*) and the ratio of renewable natural capital to physical capital (V^*) with financial development contribute a positive impact on per capita consumption c^* and subsequently on the ratio of consumption to physical capital ($U^* = c/k$). Similarly it is lead to positive impact on per capita output, y^*. In my model the development of the financial sector increases the production (production of final consumption good as well as technological capital) through accumulation of renewable natural capital. Hence the output productivity ($R^* = y/k$) is dependent on financial sector through two factors: growth of renewable natural capital and the cost of financial intermediation. While both of these factors have greater effect in early stages of development and afterwards they have vanishing effect with an increase in ϕ. Hence the output productivity (R^*) is higher for developed economies (at low ϕ) but as ϕ increases the output productivity (R^*) for developing economies grows at a faster rate as latter has vanishing effect with the increase in ϕ [Aghion et al. (2005)]. Hence financial intermediaries exert a positive but diminishing impact on output productivity and this result is also in line with existing literature [King and Levine (1993a); Beck et al. (2000); Aghion et al. (2005); Guillaumont et al. (2006)]. As a result, developing economies reach the global frontier at a faster rate by investing in financial sector [Berthelemy and Varoudakis (1996); Aghion et al. (2005); Xu et al. (2014)].

From above discussion of BGP equilibrium I can conclude that a well developed financial sector direct resources towards renewable natural resource sector where growth is driven by technology, as a result countries with well developed financial sectors display higher output growth through greater R&D intensity [Beck et al. (2000); Aghion et al. (2005), Aghion and Howitt (2008); Agn (2011); Ilyina and Samaniego (2011); Beck (2012)]. Moreover, the effect of financial development is more pronounced in developing economies and from those with high level of financial development reach to the global frontier at a faster rate which will fill the divergence gap quickly [Berthelemy and Varoudakis (1996); Aghion et al. (2005); Xu et al. (2014)].

4.2 Effects of Change in Different Parameters on BGP Equilibrium for fixed value of degree of financial development

It is clear so far that the BGP equilibrium values and economic growth is affected by financial development for both developing and developed economies. There are also other parameters in my model that affect BGP equilibrium such as share of physical capital (α), share of renewable natural capital (β), share of technological capital (χ) and inverse of intertemporal elasticity of substitution (σ). Therefore it is important to analyze the effect of change of different parameters on BGP equilibrium. I will analyze the effect of different parameters on some key variables for developing and developed economies.

4.2.1 Effects of Change in Different Parameters on BGP Equilibrium for Developed Economies at $\phi = 0.32$

The effect of change in the share of physical capital (α), share of renewable natural capital (β), share of technological capital (χ) and inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium for developed economies is presented in Table (4.1). For each parameter I have considered a low and a high value with respect to its benchmark value (refer Table (2.1)), for example the benchmark value is $\sigma = 1.5$ and I have studied the effect of inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium by taking

Table 4.1: Effect of change in parameters on BGP equilibrium for Developed Economies (at $\phi = 0.32$)

Change in parameters	R^*	U^*	V^*	W^*	z^*	x^*
$\sigma = 1.2$	0.942	0.671	13.394	1.582	0.036	0.019
$\sigma = 1.8$	0.942	0.715	5.583	1.878	0.071	0.021
$\alpha = 0.23$	1.352	1.09	10.78	2.839	0.052	0.025
$\alpha = 0.43$	0.723	0.487	5.78	1.002	0.061	0.016
$\beta = 0.21$	0.942	0.697	5.10	1.717	0.045	0.032
$\beta = 0.41$	0.942	0.697	10.59	1.464	0.064	0.013
$\chi = 0.32$	0.942	0.697	5.543	2.221	0.059	0.018
$\chi = 0.52$	0.942	0.697	9.596	1.495	0.054	0.023

$\sigma = 1.2$ (low value) and $\sigma = 1.8$ (high value).

The change in inverse of intertemporal elasticity of substitution (σ) is positively related to all BGP equilibrium except the ratio of renewable natural capital to physical capital (V^*), as higher value of inverse of intertemporal elasticity of substitution decrease the ratio of renewable natural capital to physical capital. In developed economies, the output productivity (R^*) is only affected by the change in share of physical capital (α), any given increase in α will lead to decrease in R^*, as output productivity is dependent on the utilization of renewable natural capital. Similarly it is evident that the increase in the share of physical capital will lead to decrease in ratio of consumption to physical capital (U^*), ratio of renewable natural capital to physical capital (V^*), ratio of technological capital to physical capital (W^*) as well as the extraction rate used in the production of technological capital (x^*). However the share of physical capital is positively related to aggregate extraction rate used in the production of final consumption good (z^*).

The effect of the share of renewable natural capital (β) and share of technological capital (χ) follows similar pattern, higher the share of renewable natural capital and technological capital higher will be the ratio of renewable natural capital to physical capital (V^*). However the aggregate extraction rates have a trade-off pattern due to its respective share of factor of production. The share of renewable natural is positively related to aggregate extraction rate used in the production of final consumption good, likewise the share of technological capital is positively related to aggregate extraction rate used in the production of technological capital.

4.2.2 Effects of Change in Different Parameters on BGP Equilibrium for Developing Economies at $\phi = 0.32$

The effect of change in the share of physical capital (α), share of renewable natural capital (β), share of technological capital (χ) and inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium for developing economies is presented in Table (4.2). Similar to developed economies case, for each parameter I have considered a low and a high value with respect to its benchmark value (refer Table (2.1)).

Table 4.2: Effect of change in parameters on BGP equilibrium for Developing Economies (at $\phi = 0.32$)

Change in parameters	R^*	U^*	V^*	W^*	z^*	x^*
$\sigma = 1.2$	0.692	0.482	8.185	1.087	0.034	0.013
$\sigma = 1.8$	0.692	0.502	4.172	1.320	0.053	0.012
$\alpha = 0.23$	0.993	0.773	7.818	2.074	0.042	0.015
$\alpha = 0.43$	0.531	0.345	3.872	0.655	0.048	0.010
$\beta = 0.21$	0.692	0.494	3.331	1.159	0.037	0.019
$\beta = 0.41$	0.692	0.494	7.902	1.054	0.049	0.010
$\chi = 0.32$	0.692	0.494	3.632	1.651	0.047	0.011
$\chi = 0.52$	0.692	0.494	7.010	1.004	0.043	0.014

The change in inverse of intertemporal elasticity of substitution (σ) is positively related to all BGP equilibrium except the ratio of renewable natural capital to physical capital (V^*) and extraction rate used in the production of technological capital (x^*), as higher value of inverse of intertemporal elasticity of substitution decrease the ratio of renewable natural capital to physical capital and similarly aggregate extraction rate used in the production of technological capital. However, the decrease in extraction rate used in the production of technological capital (x^*) is negligible (increase in σ from 1.2 to 1.8 will lead to 0.1 per cent decrease in x^*). Similar to developed economies case, in developing economies the output productivity (R^*) is only affected by the change in share of physical capital (α), any given increase in α will lead to decrease in R^*. Similarly it is evident that the increase in the share of physical capital will lead to decrease in ratio of consumption to physical capital (U^*), ratio of renewable natural capital to physical capital (V^*), ratio of technological capital to physical capital (W^*) as well as the extraction rate used in the production of technological capital (x^*). However the share of physical capital is positively related to aggregate extraction rate used in the production of final consumption good (z^*).

The effect of the share of renewable natural capital (β) and share of technological capital (χ) also follows similar pattern, higher the share of renewable natural capital and technological capital higher will be the ratio of renewable natural capital to physical capital (V^*). However the aggregate extraction rates have a trade-off pattern due to its respective share of factor of production. The share of renewable natural is positively related to aggregate extraction rate used in the production of final consumption good, likewise the share of technological capital is positively related to aggregate extraction rate used in the production of technological capital.

4.3 Effects of Change in Different Parameters on BGP Equilibrium with respect to change in financial development

It is clear so far that the change in the share of physical capital (α), share of renewable natural capital (β), share of technological capital (χ) and inverse of intertemporal elasticity of substitution (σ) affects BGP equilibrium for both developing and developed economies. However the effect of change in the inverse of intertemporal elasticity of substitution (σ) and the share of physical capital (α) on BGP equilibrium is more pronounced. As the given change in the share of renewable natural capital (β) and share of technological capital (χ) only affect few variables and is almost the same in both developed and developing economies. It is critical to note that the effect of change in these parameters are studied at $\phi = 0.32$. Therefore it is interesting to study the effect of change in important parameters (inverse of intertemporal elasticity of substitution (σ) and the share of physical capital (α)) on BGP equilibrium by changing financial development. I will study the effect of change in the share of physical capital (α) and inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium with respect to change in financial development for both developed and developing economies.

4.3.1 Effect of α and σ on BGP equilibrium for developed economies

The effect of change in the share of physical capital (α) and the inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium with respect to change in financial development(ϕ) is in Figure (4.3) and (4.4) respectively. In the Figure (4.3), all BGP equilibrium are negatively related to share of physical capital except the aggregate extraction of renewable natural capital used in production of final consumption good, which is positively dependent on the share of physical capital (α). It is interesting to note that, with the increase in financial development the magnitude increases. As with the increase in ϕ, any given change in the share of physical capital (α) leads to larger variation in BGP equilibrium.

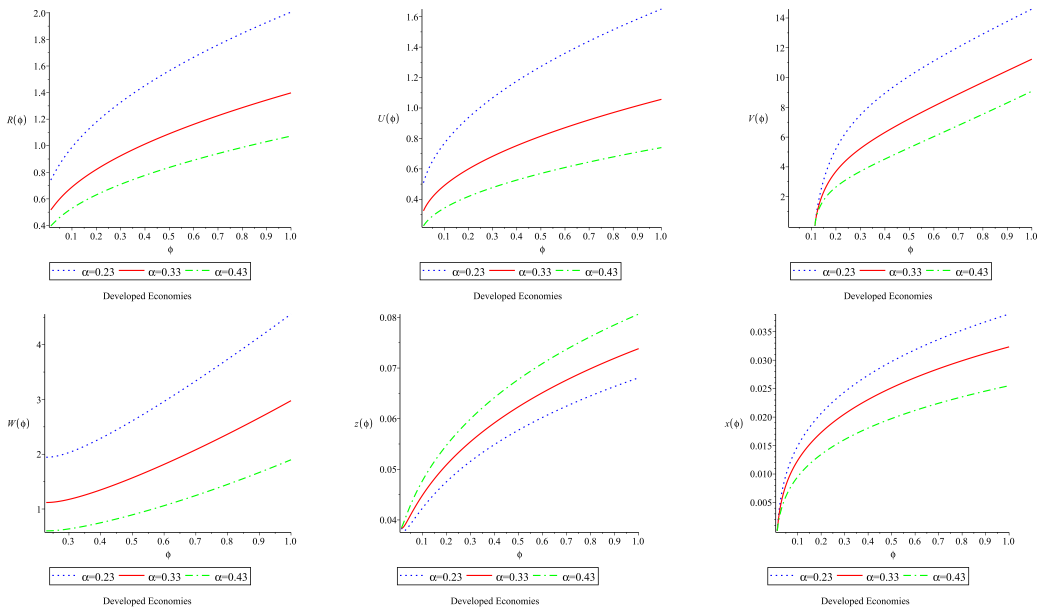

Figure 4.3: Effect of change in α on BGP equilibrium with respect of financial development in Developed Economies

The effect of the inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium is in Figure (4.4), as output productivity (R^*) is not dependent on inverse of intertemporal elasticity of substitution (σ) therefore I will only study the effect of the ratio of consumption to physical capital (U^*), ratio of renewable natural capital to physical capital (V^*), ratio of technological capital to physical capital (W^*), extraction rate used in the production of final consumption good (z^*) and used in production of technological capital (x^*). The change in inverse of intertemporal elasticity of substitution (σ) is positively related to all BGP equilibrium except the ratio of renewable natural capital to physical capital (V^*), as higher value of inverse of intertemporal elasticity of substitution (σ) decrease the ratio of renewable natural capital to physical capital. Similar to the share of physical capital, with the increase in financial development the magnitude increases. As with the increase in ϕ, any given change in inverse of intertemporal elasticity of substitution (σ) leads to larger variation in BGP equilibrium. However, the effect of inverse of intertemporal elasticity of substitution (σ) with the change in ϕ is more pronounced on the ratio of renewable natural capital to physical capital (V^*) and extraction rate used in the production of final consumption good (z^*).

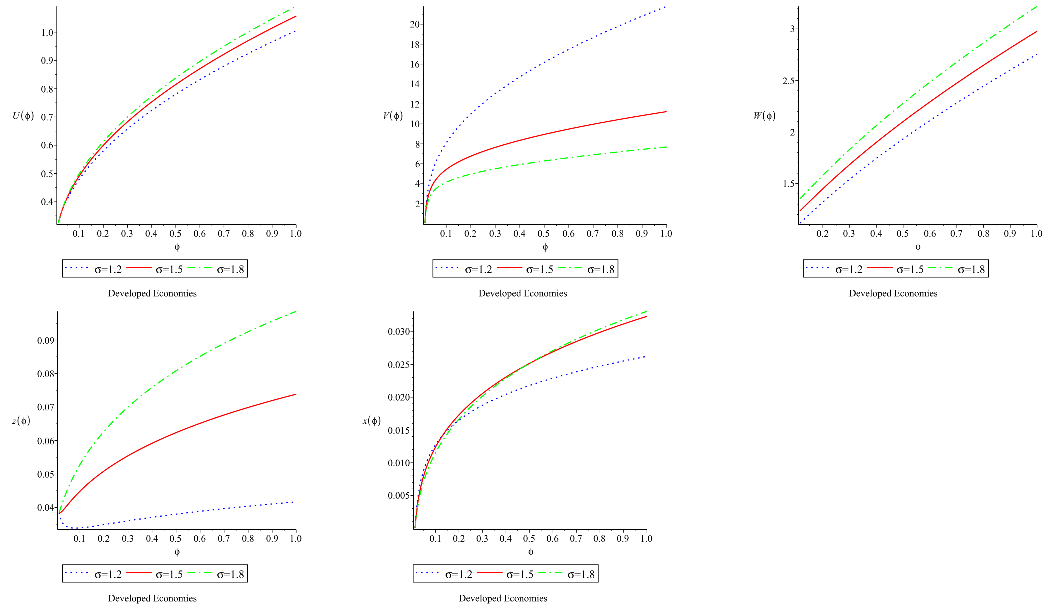

Figure 4.4: Effect of change of σ on BGP equilibrium with respect of financial development in Developed Economies

4.3.2 Effect of α and σ on BGP equilibrium for developing economies

The effect of change in the share of physical capital (α) and the inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium with respect to change in financial development(ϕ) is in Figure (4.5) and (4.6) respectively. In the Figure (4.5), all BGP equilibrium are negatively related to share of physical capital except the aggregate extraction of renewable natural capital used in production of final consumption good, which is positively dependent on the share of physical capital (α). Similar to developed economies case, with the increase in financial development the magnitude increases. As with the increase in ϕ, any given change in the share of physical capital (α) leads to larger variation in BGP equilibrium.

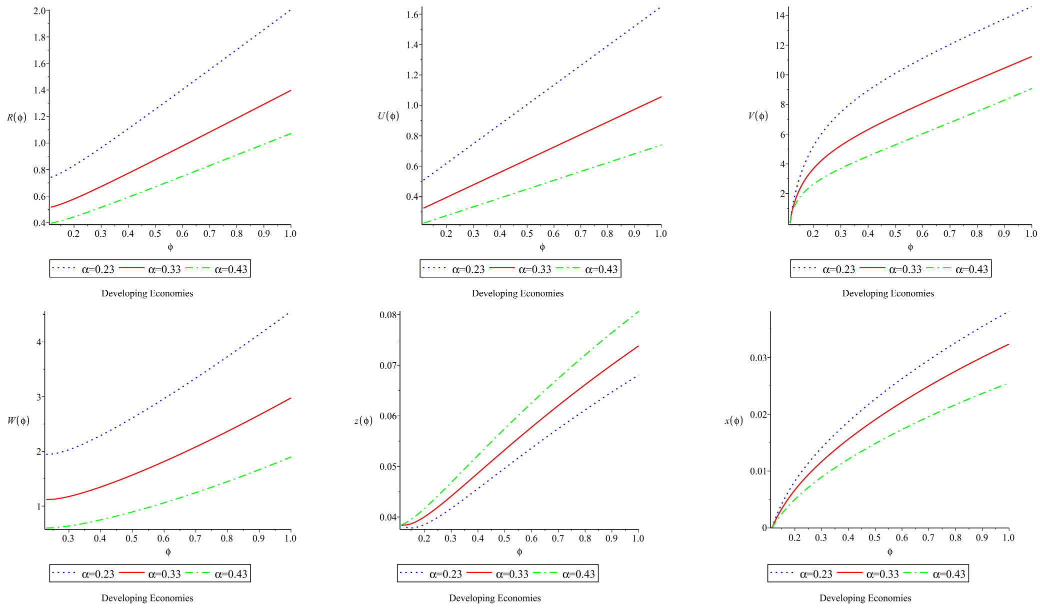

Figure 4.5: Effect of change in α on BGP equilibrium with respect of financial development in Developing Economies

The effect of the inverse of intertemporal elasticity of substitution (σ) on BGP equilibrium is in Figure (4.4), as output productivity (R^*) is not dependent on inverse of intertemporal elasticity of substitution (σ) therefore I will only study the effect of the ratio of consumption to physical capital (U^*), ratio of renewable natural capital to physical capital (V^*), the ratio of technological capital to physical capital (W^*), extraction rate used in the production of final consumption good (z^*) and used in production of technological capital (x^*). The change in inverse of intertemporal elasticity of substitution (σ) is positively related to all BGP equilibrium except the ratio of renewable natural capital to physical capital (V^*), as higher value of inverse of intertemporal elasticity of substitution (σ) decrease the ratio of renewable natural capital to physical capital. Similar to the share of physical capital (α), with the increase in financial development the magnitude increases. As with the increase in ϕ, any given change in inverse of intertemporal elasticity of substitution (σ) leads to larger variation in BGP equilibrium. However, the effect of inverse of intertemporal elasticity of substitution (σ) with the change in ϕ is more pronounced on the ratio of renewable natural capital to physical capital (V^*) and extraction rate used in the production of final consumption good (z^*).

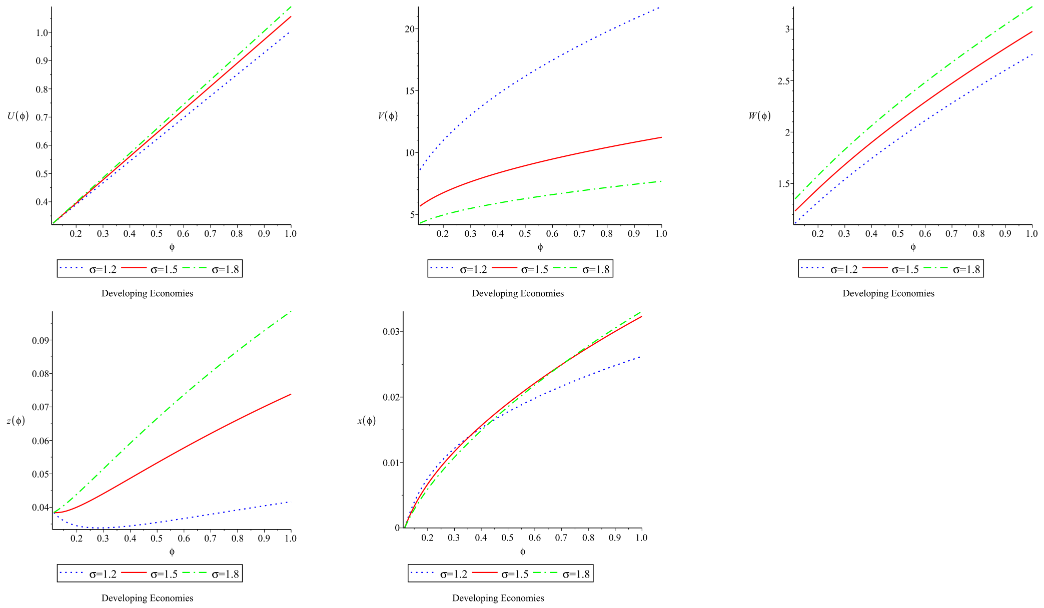

Figure 4.6: Effect of change of σ on BGP equilibrium with respect of financial development in Developing Economies

4.4 Conclusion

In this chapter I have analyzed three different effects on BGP equilibrium. Firstly I analyzed the effect of financial development on BGP equilibrium by changing the value of degree of financial development, ϕ. I found that a well developed financial sector direct resources towards renewable natural capital and increase output productivity. And the effect of financial development is more pronounced in developing economies and from those with high level of financial development reach to the global frontier at a faster rate.

Secondly, I analyzed the effect of change in key parameters on BGP equilibrium at a fixed value of degree of financial development ($\phi = 0.32$). The BGP equilibrium is affected by key parameters for both developed and developing economies.

Lastly, I analyzed the effect of change in key parameters on BGP equilibrium by changing financial development. And I found that in most the cases, the magnitude of the effect of change in key parameters on BGP equilibrium increases with the financial development. Therefore it is clear that financial development not only affects BGP equilibrium itself but it also increases the effect of change in other key parameters.

CONCLUSIONS

The thesis aims to present a finance-extended endogenous growth model to investigate the role of financial institutions on capital accumulation, output productivity and economic growth. I have formulated a three-sector endogenous growth model with constant returns to scale in renewable natural resource production in combination with physical and technological capital. In the theoretical framework, financial institutions are incorporated in the form of financial intermediaries in all three sectors to analyze how financial development impacts economic growth and output productivity through the channel of renewable natural capital. As, sound financial institutions improve savings and investments and also effectively allocate resources in capital producing ventures that in return enhance output productivity and economic growth.

Financial intermediaries impact all three sectors renewable natural capital, physical capital and technological capital by effecting the growth rate of renewable natural capital $B(\phi)$, cost of financial intermediation $\xi(\phi)$, and the scale parameter of technology $\nu(\phi)$ respectively. The growth rate of renewable natural capital and the scale parameter of technology is positively related to financial development, higher the ϕ higher will be the value of $B(\phi)$ and $\nu(\phi)$. However, with the increase in financial development the cost of financial intermediation decreases hence a negative relationship exists between ϕ and $\xi(\phi)$. Moreover it is worth noting that the effect of financial development is high in early stages of development and afterwards it has vanishing effect. Therefore with an increase in ϕ, the growth rate of renewable natural capital ($B(\phi)$) and the scale parameter of technology ($\nu(\phi)$) grow at a faster rate for developing economies. Likewise, the cost of financial intermediation ($\xi(\phi)$) decreases at a faster rate for developing economies. These results are also inline with existing literature [Fung (2009); Xu et al. (2014); Philippon (2015)].

Inline with King and Levine (1993a), Beck et al. (2000) and Rioja and

Valev (2004), I found that financial development is positively and monotonically related to economic growth. The growth rate of renewable natural capital ($B(\phi)$) is an important determinant through which financial development positively contribute to economic growth. Renewable natural capital accumulation heavily rely on external financing so increasing financial deepening not only ease the credit channel but also enhance the effectiveness of credit [Brunnschweiler (2010); Kim and Park (2016)]. Furthermore, this finding also support the empirical literature [Saidi (2006); Ba et al. (2010); Calitz and Fourie (2010); Mathews et al. (2010); Scholtens and Veldhuis (2015)] where it is argued that economic growth is a key determinant to private investment in power projects and the decision of private investors is largely influenced by country's financial institution quality. The effect of financial development on BGP equilibrium is also positive i.e. monotonic for both developing and developed economies whereas in the case of developed economies financial development has vanishing effect as suggested by Rousseau and Wachtel (2011) and later by Arcand et al. (2015).

In this model the development of the financial sector increases the production (production of final consumption good as well as technological capital) through accumulation of renewable natural capital. Hence the output productivity ($R^* = y/k$) is dependent on financial sector through two factors: growth of renewable natural capital and the cost of financial intermediation. While both of these factors have greater effect in early stages of development and afterwards they have vanishing effect with an increase in ϕ. Hence the output productivity (R^*) increases at a faster rate for developing economies with respect to developed economies as latter has vanishing effect. Hence financial intermediaries exert a positive but diminishing impact on output productivity and this result is also in line with existing literature [King and Levine (1993a); Beck et al. (2000); Aghion et al. (2005); Guillaumont et al. (2006)]. As a result, developing economies with high level of financial development reach global frontier at a faster rate which will fill the divergence gap quickly [Berthelemy and Varoudakis (1996); Aghion et al. (2005); Xu et al. (2014); Ilyina and Samaniego (2011)].

In my model, I have found that finance have a positive but vanishing effect on economic growth and output productivity as suggested by Aghion et al. (2005) that the effect of financial development diminishes as economy moves towards the global frontier. However my results differ from Bucci and Marsiglio (2018) where they found a non-monotonic (inverted U-shaped) relationship of financial development and economic growth exists when the depreciation effect of human capital off sets the productivity effect of human capital. The critical assumption in their model is the relationship of depreciation with financial development as depreciation is a function of financial development which is not necessarily true in every sector. Lastly in their model the non-monotonic relationship exists only under specific functional forms of productivity and depreciation of human capital. As the monotonic relationship exists under linear functional forms of productivity and depreciation of human capital and non-monotonic under exponential and quadratic forms of productivity and depreciation of human capital. In my model, economic growth is only affected by the productivity effect of renewable natural capital as there is no depreciation effect. Hence I would suggest that more theoretical research is necessary to study finance-growth relationship. In this regards, future research can be done by including an externality in renewable natural resource sector or by incorporating non-renewable natural resource sector in the model.

To sum up all, a well developed financial sector direct resources towards renewable natural resource sector where growth is driven by technology, as a result countries with well developed financial sector display higher output growth [Beck et al. (2000); Aghion et al. (2005), Aghion and Howitt (2008); Agn (2011); Ilyina and Samaniego (2011); Beck (2012)]. And the effect of financial development diminishes as economy moves towards the global frontier, therefore developing economies with high level of financial development reach global frontier at a faster rate which will fill the divergence gap quickly [Berthelemy and Varoudakis (1996); Aghion et al. (2005); Xu et al. (2014); Ilyina and Samaniego (2011)]. Hence, I can conclude that policy makers in developing economies should focus on their financial institutions in order to enhance output productivity and economic growth through investment in renewable natural capital.

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Appendix A

As mentioned in Chapter 1, the solution of the model does not exist for the original variables so I will use dimensionality reduction technique by taking ratios of variables. Therefore I can study the dynamics of simplified system by introducing the variables: $U = \frac{c}{k}$, $V = \frac{q}{k}$ and $W = \frac{A}{k}$. The time derivatives of these variables are as follows:

$$\frac{\dot{U}}{U} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} \tag{A-1}$$

$$\frac{\dot{V}}{V} = \frac{\dot{q}}{q} - \frac{\dot{k}}{k} \tag{A-2}$$

$$\frac{\dot{W}}{W} = \frac{\dot{A}}{A} - \frac{\dot{k}}{k} \tag{A-3}$$

With the aid of equations (1.46)-(1.48) and (1.57), I can rewrite equations (A-1), (A-2) and (A-3) as follows:

$$\frac{\dot{U}}{U} = \frac{1}{\sigma}\left[(\alpha - \sigma)(1 - \xi(\phi))k^{\alpha-1}(zq)^{\beta}A^{1-\alpha-\beta} - (1-\sigma)\delta - \rho + \sigma\frac{c}{k}\right] \tag{A-4}$$

$$\frac{\dot{V}}{V} = B(\phi)(1 - z - x) - z - x - (1 - \xi(\phi))k^{\alpha}(zq)^{\beta}A^{1-\alpha-\beta} + \delta + \frac{c}{k} \tag{A-5}$$

$$\frac{\dot{W}}{W} = \nu(\phi)\left(\frac{xq}{A}\right)^{\chi} - (1 - \xi(\phi))k^{\alpha}(zq)^{\beta}A^{1-\alpha-\beta} + \delta + \frac{c}{k} \tag{A-6}$$

By using the variables $U = \frac{c}{k}$, $V = \frac{q}{k}$ and $W = \frac{A}{k}$ in equations (A-4), (A-5), (A-6), (1.58) and (1.59) I can derive following system of five nonlinear differential equations:

$$\frac{\dot{U}}{U} = \frac{1}{\sigma}\left[(\alpha - \sigma)(1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} - (1-\sigma)\delta - \rho + \sigma U\right] \tag{A-7}$$

$$\frac{\dot{V}}{V} = B(\phi)(1 - z - x) - z - x - (1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} + \delta + U \tag{A-8}$$

$$\frac{\dot{W}}{W} = \nu(\phi)\left(\frac{xV}{W}\right)^{\chi} - (1 - \xi(\phi))(zV)^{\beta}W^{1-\alpha-\beta} + \delta + U \tag{A-9}$$

$$\frac{\dot{z}}{z} = \frac{1}{\beta - 1}\Bigg[\alpha U - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)\left(\frac{xV}{W}\right)^{\chi} - (1-\beta)(B(\phi)+1)(z+x)\Bigg], \tag{A-10}$$

$$\frac{\dot{x}}{x} = \frac{1}{\chi - 1}\Bigg[\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z}{x}\left(\frac{xV}{W}\right)^{\chi} - \chi B(\phi) - (1-\chi)(B(\phi)+1)(z+x)\Bigg]. \tag{A-11}$$

Moreover, by introducing the variables $R = (zV)^{\beta}W^{1-\alpha-\beta}$ and $S = \left(\frac{xV}{W}\right)^{\chi}$, I can rewrite the system as follows:

$$\frac{\dot{U}}{U} = \frac{1}{\sigma}\Big[(\alpha-\sigma)(1-\xi(\phi))R - (1-\sigma)\delta - \rho + \sigma U\Big] \tag{A-12}$$

$$\frac{\dot{V}}{V} = B(\phi)(1-z-x) - z - x - (1-\xi(\phi))R + \delta + U \tag{A-13}$$

$$\frac{\dot{W}}{W} = \nu(\phi)S - (1-\xi(\phi))R + \delta + U \tag{A-14}$$

$$\frac{\dot{z}}{z} = \frac{1}{\beta - 1}\Big[\alpha U - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)S - (1-\beta)(B(\phi)+1)(z+x)\Big] \tag{A-15}$$

$$\frac{\dot{x}}{x} = \frac{1}{\chi - 1}\Big[\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z}{x}S - \chi B(\phi) - (1-\chi)(B(\phi)+1)(z+x)\Big] \tag{A-16}$$

The time derivatives of the variables $R = (zV)^{\beta}W^{1-\alpha-\beta}$ and $S = \left(\frac{xV}{W}\right)^{\chi}$ are as follows:

$$\frac{\dot{R}}{R} = \beta\left(\frac{\dot{z}}{z} + \frac{\dot{V}}{V}\right) + (1-\alpha-\beta)\frac{\dot{W}}{W} \tag{A-17}$$

$$\frac{\dot{S}}{S} = \chi\left(\frac{\dot{x}}{x} + \frac{\dot{V}}{V} - \frac{\dot{W}}{W}\right) \tag{A-18}$$

further can be written as follows:

$$\frac{\dot{R}}{R} = \left(\frac{\alpha+\beta-1}{\beta-1}\right)U - \left(\frac{\beta}{\beta-1}\right)B(\phi) - (1-\alpha)(1-\xi(\phi))R - \left(\frac{1-\alpha-\beta}{\beta-1}\right)\nu(\phi)S - \left(\frac{1-\alpha}{\beta-1}\right)\delta \tag{A-19}$$

$$\frac{\dot{S}}{S} = \frac{\chi^2\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}\frac{z}{x}S - \chi\nu(\phi)S - \left(\frac{\chi}{\chi-1}\right)B(\phi) \tag{A-20}$$

Appendix B

The steady state solution of five dynamical equations system can be found by setting (2.6)-(2.10) equal to zero:

$$0 = (\alpha - \sigma)(1 - \xi(\phi))R^* - (1 - \sigma)\delta - \rho + \sigma U^* \tag{B-1}$$

$$0 = B(\phi)(1 - z^* - x^*) - z^* - x^* - (1 - \xi(\phi))R^* + \delta + U^* \tag{B-2}$$

$$0 = \nu(\phi)S^* - (1 - \xi(\phi))R^* + \delta + U^* \tag{B-3}$$

$$\begin{aligned} 0 = \alpha U^* - (1 - \alpha)\delta - \beta B(\phi) - (1 - \alpha - \beta)\nu(\phi)S^* \\ -(1 - \beta)(B(\phi) + 1)(z^* + x^*) \end{aligned} \tag{B-4}$$

$$0 = \frac{\chi\nu(\phi)(1 - \alpha - \beta)}{\beta}\frac{z^*}{x^*}S^* - \chi B(\phi) - (1 - \chi)(B(\phi) + 1)(z^* + x^*) \tag{B-5}$$

By plugging equation (B-2) into equation (B-3) I will obtain,

$$\nu(\phi)S^* - B(\phi)(1 - z^* - x^*) + z^* + x^* = 0 \tag{B-6}$$

$$\nu(\phi)S^* - B(\phi) + B(\phi)z^* + B(\phi)x^* + z^* + x^* = 0 \tag{B-7}$$

$$(B(\phi) + 1)(z^* + x^*) = B(\phi) - \nu(\phi)S^* \tag{B-8}$$

substituting equation (B-8) into equation (B-4) I will get,

$$\alpha U^* - (1 - \alpha)\delta - \beta B(\phi) - (1 - \alpha - \beta)\nu(\phi)S^* - (1 - \beta)(B(\phi) - \nu(\phi)S^*) = 0 \tag{B-9}$$

$$\alpha U^* = B(\phi) - \alpha\nu(\phi)S^* + (1 - \alpha)\delta \tag{B-10}$$

$$U^* = \frac{B(\phi)}{\alpha} - \nu(\phi)S^* + \frac{(1 - \alpha)}{\alpha}\delta \tag{B-11}$$

substituting value from equation (B-11) into equation (B-1):

$$(\alpha - \sigma)(1 - \xi(\phi))R^* - (1 - \sigma)\delta - \rho + \frac{\sigma}{\alpha}B(\phi) - \sigma\nu(\phi)S^* + \frac{\sigma(1 - \alpha)}{\alpha}\delta = 0 \tag{B-12}$$

$$R^* = \frac{1}{(\alpha - \sigma)(1 - \xi(\phi))}\left[\rho - \frac{\sigma}{\alpha}B(\phi) + \sigma\nu(\phi)S^* + \frac{\alpha - \sigma}{\alpha}\delta\right] \tag{B-13}$$

Now, by plugging value from equations (B-11)and (B-13) into equation (B-3), I can find steady state value of S^*:

$$\nu(\phi)S^* - \frac{1}{\alpha-\sigma}\left[\rho - \frac{\sigma}{\alpha}B(\phi) + \sigma\nu(\phi)S^* + \frac{\alpha-\sigma}{\alpha}\delta\right] + \frac{B(\phi)}{\alpha} - \nu(\phi)S^* + \frac{(1-\alpha)}{\alpha}\delta + \delta = 0 \tag{B-14}$$

$$\frac{1}{\alpha-\sigma}\left[B(\phi) - \rho - \sigma\nu(\phi)S^*\right] = 0 \tag{B-15}$$

$$S^* = \frac{B(\phi)-\rho}{\sigma\nu(\phi)} \tag{B-16}$$

by using value from equation (B-16), equation (B-11) and (B-13) can be written as:

$$U^* = \frac{B(\phi)}{\alpha} - \nu(\phi)\frac{B(\phi)-\rho}{\sigma\nu(\phi)} + \frac{(1-\alpha)}{\alpha}\delta \tag{B-17}$$

$$U^* = \frac{B(\phi)\sigma - \alpha(B(\phi)-\rho)}{\alpha\sigma} + \frac{(1-\alpha)}{\alpha}\delta \tag{B-18}$$

$$R^* = \frac{1}{(\alpha-\sigma)(1-\xi(\phi))}\left[\rho - \frac{\sigma}{\alpha}B(\phi) + \sigma\nu(\phi)\frac{B(\phi)-\rho}{\sigma\nu(\phi)} + \frac{\alpha-\sigma}{\alpha}\delta\right] \tag{B-19}$$

$$= \frac{1}{(\alpha-\sigma)(1-\xi(\phi))}\left[\frac{\alpha-\sigma}{\alpha}B(\phi) + \frac{\alpha-\sigma}{\alpha}\delta\right] \tag{B-20}$$

$$R^* = \frac{B(\phi)+\delta}{\alpha(1-\xi(\phi))} \tag{B-21}$$

Now, with the aid of equation (B-5) and (B-8) , I will get,

$$\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z^*}{x^*}S^* - \chi B(\phi) - (1-\chi)(B(\phi) - \nu(\phi)S^*) = 0 \tag{B-22}$$

$$\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta}\frac{z^*}{x^*}S^* = B(\phi) - (1-\chi)\nu(\phi)S^* \tag{B-23}$$

substituting value of S^* from equation (B-16)

$$\frac{\chi(1-\alpha-\beta)}{\beta}\frac{z^*}{x^*}\left(\frac{B(\phi)-\rho}{\sigma}\right) = B(\phi) - (1-\chi)\left(\frac{B(\phi)-\rho}{\sigma}\right) \tag{B-24}$$

$$\chi(1-\alpha-\beta)(B(\phi)-\rho)z^* = \beta(\sigma B(\phi) - (1-\chi)(B(\phi)-\rho))x^* \tag{B-25}$$

$$z^* = \frac{\beta\chi(B(\phi)-\rho) + \beta(\sigma B(\phi) + \rho - B(\phi))}{\chi(1-\alpha-\beta)(B(\phi)-\rho)}x^* \tag{B-26}$$

by plugging value from (B-16) and (B-26) into equation (B-8) I can compute steady state value of x^*:

$$(B(\phi)+1)\left[\frac{\beta\chi(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))}{\chi(1-\alpha-\beta)(B(\phi)-\rho)}x^*+x^*\right] = B(\phi)-\frac{B(\phi)-\rho}{\sigma} \tag{B-27}$$

$$x^*(B(\phi)+1)\left[\frac{\beta\chi(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))}{\chi(1-\alpha-\beta)(B(\phi)-\rho)}+1\right] = \frac{\sigma B(\phi)+\rho-B(\phi)}{\sigma} \tag{B-28}$$

$$x^*(B(\phi)+1)\left[\frac{\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))}{\chi(1-\alpha-\beta)(B(\phi)-\rho)}\right] = \frac{\sigma B(\phi)+\rho-B(\phi)}{\sigma} \tag{B-29}$$

$$x^* = \frac{\chi(1-\alpha-\beta)(B(\phi)-\rho)(\sigma B(\phi)+\rho-B(\phi))}{\sigma(B(\phi)+1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]} \tag{B-30}$$

substituting the value of x^* from equation (B-30) in equation (B-26) yields:

$$z^* = \left[\frac{\beta\chi(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))}{\chi(1-\alpha-\beta)(B(\phi)-\rho)}\right] \left[\frac{\chi(1-\alpha-\beta)(B(\phi)-\rho)(\sigma B(\phi)+\rho-B(\phi))}{\sigma(B(\phi)+1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]}\right] \tag{B-31}$$

$$z^* = \frac{(\sigma B(\phi)+\rho-B(\phi))[\beta\chi(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]}{\sigma(B(\phi)+1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta(\sigma B(\phi)+\rho-B(\phi))]} \tag{B-32}$$

Lastly, I can derive the values of V and W by using the variables $R^* = (z^*V^*)^\beta W^{*1-\alpha-\beta}$ and $S^* = \left(\frac{x^*V^*}{W^*}\right)^\chi$:

$$V^* = \left(\frac{x^{*\alpha+\beta-1}R^*}{z^{*\beta}S^{*\frac{\alpha+\beta-1}{\chi}}}\right)^{\frac{1}{1-\alpha}} \tag{B-33}$$

$$W^* = \left(\frac{x^*R^{*\frac{1}{\beta}}}{z^*S^{*\frac{1}{\chi}}}\right)^{\frac{\beta}{1-\alpha}} \tag{B-34}$$

Appendix C

I have build Jacobian matrix to study the local stability of the steady state equilibrium. The Jacobian matrix, $J(U, R, S, z, x)$ is:

$$J = \begin{bmatrix} \frac{\partial \dot{U}}{\partial U} & \frac{\partial \dot{U}}{\partial R} & \frac{\partial \dot{U}}{\partial S} & \frac{\partial \dot{U}}{\partial z} & \frac{\partial \dot{U}}{\partial x} \\ \frac{\partial \dot{R}}{\partial U} & \frac{\partial \dot{R}}{\partial R} & \frac{\partial \dot{R}}{\partial S} & \frac{\partial \dot{R}}{\partial z} & \frac{\partial \dot{R}}{\partial x} \\ \frac{\partial \dot{S}}{\partial U} & \frac{\partial \dot{S}}{\partial R} & \frac{\partial \dot{S}}{\partial S} & \frac{\partial \dot{S}}{\partial z} & \frac{\partial \dot{S}}{\partial x} \\ \frac{\partial \dot{z}}{\partial U} & \frac{\partial \dot{z}}{\partial R} & \frac{\partial \dot{z}}{\partial S} & \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial x} \\ \frac{\partial \dot{x}}{\partial U} & \frac{\partial \dot{x}}{\partial R} & \frac{\partial \dot{x}}{\partial S} & \frac{\partial \dot{x}}{\partial z} & \frac{\partial \dot{x}}{\partial x} \end{bmatrix}$$

By using the five dynamical system of equations (2.6)-(2.10) I calculated elements or entries of Jacobian matrix, J, as follows:

$$\frac{\partial \dot{U}}{\partial U} = \frac{\alpha - \sigma}{\sigma}(1 - \xi(\phi))R - \frac{1 - \sigma}{\sigma}\delta - \frac{\rho}{\sigma} + 2U \tag{C-1}$$

$$\frac{\partial \dot{U}}{\partial R} = \frac{\alpha - \sigma}{\sigma}(1 - \xi(\phi))U \tag{C-2}$$

$$\frac{\partial \dot{U}}{\partial S} = 0 \tag{C-3}$$

$$\frac{\partial \dot{U}}{\partial z} = 0 \tag{C-4}$$

$$\frac{\partial \dot{U}}{\partial x} = 0 \tag{C-5}$$

$$\frac{\partial \dot{R}}{\partial U} = \frac{\alpha + \beta - 1}{\beta - 1}R \tag{C-6}$$

$$\begin{aligned} \frac{\partial \dot{R}}{\partial R} = \frac{\alpha + \beta - 1}{\beta - 1}U - \frac{\beta}{\beta - 1}B(\phi) - 2(1 - \alpha)(1 - \xi(\phi))R \\ - \frac{1 - \alpha - \beta}{\beta - 1}\nu(\phi)S - \frac{1 - \alpha}{\beta - 1}\delta \end{aligned} \tag{C-7}$$

$$\frac{\partial \dot{R}}{\partial S} = -\frac{1 - \alpha - \beta}{\beta - 1}\nu(\phi)R \tag{C-8}$$

$$\frac{\partial \dot{R}}{\partial z} = 0 \tag{C-9}$$

$$\frac{\partial \dot{R}}{\partial x} = 0 \tag{C-10}$$

$$\frac{\partial \dot{S}}{\partial U} = 0 \tag{C-11}$$

$$\frac{\partial \dot{S}}{\partial R} = 0 \tag{C-12}$$

$$\frac{\partial \dot{S}}{\partial S} = \frac{2\chi^2\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}\frac{z}{x}S - 2\chi\nu(\phi)S - \frac{\chi}{\chi-1}B(\phi) \tag{C-13}$$

$$\frac{\partial \dot{S}}{\partial z} = \frac{\chi^2\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}\frac{S^2}{x} \tag{C-14}$$

$$\frac{\partial \dot{S}}{\partial x} = -\frac{\chi^2\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}\frac{z}{x^2}S^2 \tag{C-15}$$

$$\frac{\partial \dot{z}}{\partial U} = \frac{\alpha}{\beta-1}z \tag{C-16}$$

$$\frac{\partial \dot{z}}{\partial R} = 0 \tag{C-17}$$

$$\frac{\partial \dot{z}}{\partial S} = -\frac{(1-\alpha-\beta)\nu(\phi)}{\beta-1}z \tag{C-18}$$

$$\begin{aligned}\frac{\partial \dot{z}}{\partial z} = \frac{1}{\beta-1}\Big[\alpha U - (1-\alpha)\delta - \beta B(\phi) - (1-\alpha-\beta)\nu(\phi)S\Big] \\ +(B(\phi)+1)(2z+x)\end{aligned} \tag{C-19}$$

$$\frac{\partial \dot{z}}{\partial x} = (B(\phi)+1)z \tag{C-20}$$

$$\frac{\partial \dot{x}}{\partial U} = 0 \tag{C-21}$$

$$\frac{\partial \dot{x}}{\partial R} = 0 \tag{C-22}$$

$$\frac{\partial \dot{x}}{\partial S} = \frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}z \tag{C-23}$$

$$\frac{\partial \dot{x}}{\partial z} = \frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)}S + (B(\phi)+1)x \tag{C-24}$$

$$\frac{\partial \dot{x}}{\partial x} = -\frac{\chi}{\chi-1}B(\phi) + (B(\phi)+1)(z+2x) \tag{C-25}$$

whereas at the steady state I can write the Jacobian matrix,$J^*(U^*, R^*, S^*, z^*, x^*)$ as follows:

$$J^* = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

$$a_{11} = U^* \tag{C-26}$$

$$a_{12} = \frac{\alpha - \sigma}{\sigma}(1 - \xi(\phi))U^* \tag{C-27}$$

$$a_{13} = 0 \tag{C-28}$$

$$a_{14} = 0 \tag{C-29}$$

$$a_{15} = 0 \tag{C-30}$$

$$a_{21} = \frac{\alpha + \beta - 1}{\beta - 1}R^* \tag{C-31}$$

$$a_{22} = -(1 - \alpha)(1 - \xi(\phi))R^* \tag{C-32}$$

$$a_{23} = -\frac{1 - \alpha - \beta}{\beta - 1}\nu(\phi)R^* \tag{C-33}$$

$$a_{24} = 0 \tag{C-34}$$

$$a_{25} = 0 \tag{C-35}$$

$$a_{31} = 0 \tag{C-36}$$

$$a_{32} = 0 \tag{C-37}$$

$$a_{33} = \frac{\chi^2\nu(\phi)(1 - \alpha - \beta)}{\beta(\chi - 1)}\frac{z^*}{x^*}S^* - \chi\nu(\phi)S^* \tag{C-38}$$

$$a_{34} = \frac{\chi^2\nu(\phi)(1 - \alpha - \beta)}{\beta(\chi - 1)}\frac{S^{*2}}{x^*} \tag{C-39}$$

$$a_{35} = -\frac{\chi^2\nu(\phi)(1 - \alpha - \beta)}{\beta(\chi - 1)}\frac{z^*}{x^{*2}}S^{*2} \tag{C-40}$$

$$a_{41} = \frac{\alpha}{\beta - 1}z^* \tag{C-41}$$

$$a_{42} = 0 \tag{C-42}$$

$$a_{43} = -\frac{(1-\alpha-\beta)\nu(\phi)}{\beta - 1} z^* \tag{C-43}$$

$$a_{44} = (B(\phi) + 1) z^* \tag{C-44}$$

$$a_{45} = (B(\phi) + 1) z^* \tag{C-45}$$

$$a_{51} = 0 \tag{C-46}$$

$$a_{52} = 0 \tag{C-47}$$

$$a_{53} = \frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)} z^* \tag{C-48}$$

$$a_{54} = \frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)} S^* + (B(\phi)+1)x^* \tag{C-49}$$

$$a_{55} = -\frac{\chi\nu(\phi)(1-\alpha-\beta)}{\beta(\chi-1)} \frac{z^*}{x^*} S^* + (B(\phi)+1)x^* \tag{C-50}$$

by plugging steady states values from (2.11)-(2.15), I can rewrite the Jacobian matrix,$J^*(U^*, R^*, S^*, z^*, x^*)$ as follows:

$$\begin{bmatrix} \frac{(\sigma-\alpha)B(\phi)+\alpha\rho+\delta(\sigma-\sigma\alpha)}{\sigma\alpha} & \frac{(\alpha-\sigma)(1-\xi(\phi))((\sigma-\alpha)B(\phi)-\alpha\rho-\delta(\sigma-\sigma\alpha))}{\alpha\sigma^2} \\ \frac{(-1+\alpha+\beta)(B(\phi)+\delta)}{\alpha(\beta-1)(1-\xi(\phi))} & \frac{(\alpha-1)(B(\phi)+\delta)}{\alpha} \\ 0 & 0 \\ \frac{\alpha\beta(\sigma B(\phi)-B(\phi)+\rho)(\chi B(\phi)+\sigma B(\phi)-B(\phi)-\chi\rho+\rho)}{(\beta-1)\sigma(B(\phi)+1)[B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi)]} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \frac{(-1+\alpha+\beta)\nu(\phi)(B(\phi)+\delta)}{(\beta-1)\alpha(1-\xi(\phi))} \\ \frac{\chi B(\phi)}{\chi-1} \\ -\frac{(-1+\alpha+\beta)\nu(\phi)\beta((\sigma-1)B(\phi)+\rho)((\chi+\sigma-1)B(\phi)+\rho(1-\chi))}{(\beta-1)\sigma(B(\phi)+1)[B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi)]} \\ \frac{(-1+\alpha+\beta)\nu(\phi)\chi((\sigma-1)B(\phi)+\rho)(\chi+\sigma-1)B(\phi)+\rho(1-\chi)}{(\beta-1)\sigma(B(\phi)+1)[B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi)]} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -\frac{\chi(B(\phi)-\rho)(B(\phi)+1)[B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi)]}{\sigma\nu(\phi)\beta(\chi-1)((\sigma-1)B(\phi)+\rho)} \\ -\frac{\beta((\sigma-1)B(\phi)+\rho)[\chi(B(\phi)-\rho)+(\sigma-1)B(\phi)]}{\sigma(B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi))} \\ -\frac{\chi^2(-1+\alpha+\beta)(B(\phi)-\rho)[(B(\phi)-\rho)(\alpha-1)-\beta((\sigma-1)B(\phi)+\rho)}{\sigma\beta(\chi-1)(B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi))} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -\frac{(B(\phi)+1)[\chi(B(\phi)-\rho)+(\sigma-1)B(\phi)+\rho](B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi))}{\sigma\nu(\phi)(-1+\alpha+\beta)(\chi-1)(B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi))} \\ -\frac{\beta((\sigma-1)B(\phi)+\rho)[\chi(B(\phi)-\rho)+(\sigma-1)B(\phi)+\rho]}{\sigma(B(\phi)(\alpha\chi-\beta\sigma+\beta-\chi)-\rho(\alpha\chi+\beta-\chi))} \\ \frac{(((1-\alpha-\beta)\sigma+\beta+2\alpha-2)\chi^2+(2(\sigma-1))(-1+\alpha)\chi-\beta(\sigma-1)^2)B(\phi)^2}{\sigma(\chi-1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta((\sigma-1)B(\phi)+\rho)]} \\ +\frac{(((\alpha+\beta-1)\sigma-2\beta-4\alpha+4)\chi^2-(2(\sigma-2))(-1+\alpha)\chi-2\beta(\sigma-1))\rho B(\phi)+\rho^2((\beta+2\alpha-2)\chi+\beta)(\chi-1)}{\sigma(\chi-1)[\chi(1-\alpha)(B(\phi)-\rho)+\beta((\sigma-1)B(\phi)+\rho)]} \end{bmatrix}$$